SHORT CIRCUIT CALCULATIONS REVISITED

FORTUNATO C. LEYNES

Chairman

Professional Regulatory

Board of Electrical Engineering

Professional Regulation Commission

Short Circuit (Fault) Analysis

- *FAULT-PROOF SYSTEM*
  - *not practical*
  - *neither economical*
  - *faults or failures occur in any power system*

- *In the various parts of the electrical network under short circuit or unbalanced condition, the determination of the magnitudes and phase angles*
  - *Currents*
  - *Voltages*
  - *Impedances*

Application of Fault Analysis

1. The determination of the required mechanical strength of electrical equipment to withstand the stresses brought about by the flow of high short circuit currents
2. The selection of circuit breakers and switch ratings
3. The selection of protective relay and fuse ratings

Application of Fault Analysis

4. The setting and coordination of protective devices
5. The selection of surge arresters and insulation ratings of electrical equipment
6. The determination of fault impedances for use in stability studies
7. The calculation of voltage sags caused resulting from short circuits

Application of Fault Analysis

8. The sizing of series reactors to limit the short circuit current to a desired value
9. To determine the short circuit capability of series capacitors used in series compensation of long transmission lines
10. To determine the size of grounding transformers, resistances, or reactors

Per Unit Calculations

Three-phase Systems

$$Z_B = \frac{(base\ voltage,\ \ kV_{L-L})^2 \times 1000}{base\ kVA_{3\Phi}}$$

$$Z_B = \frac{(base\ voltage,\ \ kV_{L-L})^2}{base\ MVA_{3\Phi}}$$

Per Unit Quantities

$$I_{pu} = \frac{actual\ current}{Base\ Current\ (I_B)}$$

$$V_{pu} = \frac{actual\ voltage\ (kV)}{Base\ Voltage\ (kV_B)}$$

$$Z_{pu} = \frac{actual\ impedance}{Base\ impedance\ (Z_B)}$$

Changing the Base of Per Unit Quantities

$$Z_{pu[old]} = \frac{actual\ impedance,\ Z(\Omega)}{\dfrac{\left(base\ kV_{[old]}\right)^2 \times 1000}{base\ kVA_{[old]}}}$$

$$Z(\Omega) = \frac{Z_{pu[old]}\left(base\ kV_{[old]}\right)^2 \times 1000}{base\ kVA_{[old]}}$$

$$Z_{B[new]} = \frac{\left(base\ kV_{[new]}\right)^2 \times 1000}{base\ kVA_{[new]}}$$

$$Z_{pu[new]} = \frac{Z(\Omega)}{Z_{B[new]}}$$

Changing the Base of Per Unit Quantities

$$Z_{pu[old]} = \frac{actual\ impedance,\ Z(\Omega)}{\dfrac{\left(base\ kV_{[old]}\right)^2 \times 1000}{base\ kVA_{[old]}}}$$

$$Z(\Omega) = \frac{Z_{pu[old]}\left(base\ kV_{[old]}\right)^2 \times 1000}{base\ kVA_{[old]}}$$

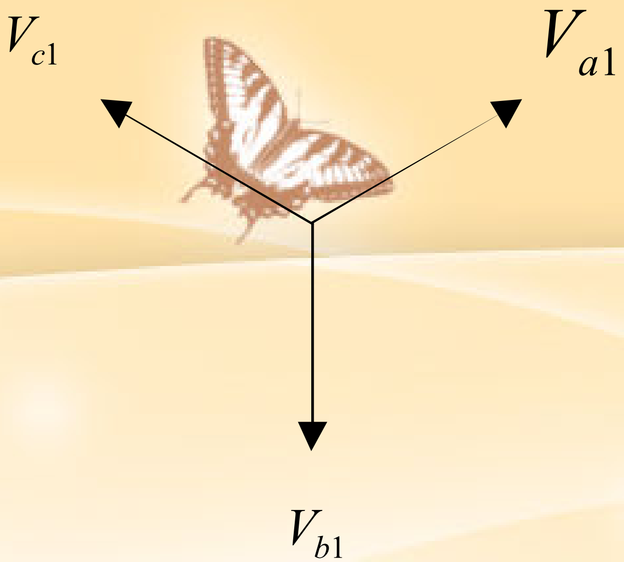
$$Z_{B[new]} = \frac{\left(base\ kV_{[new]}\right)^2 \times 1000}{base\ kVA_{[new]}}$$

$$Z_{pu[new]} = \frac{Z(\Omega)}{Z_{B[new]}}$$

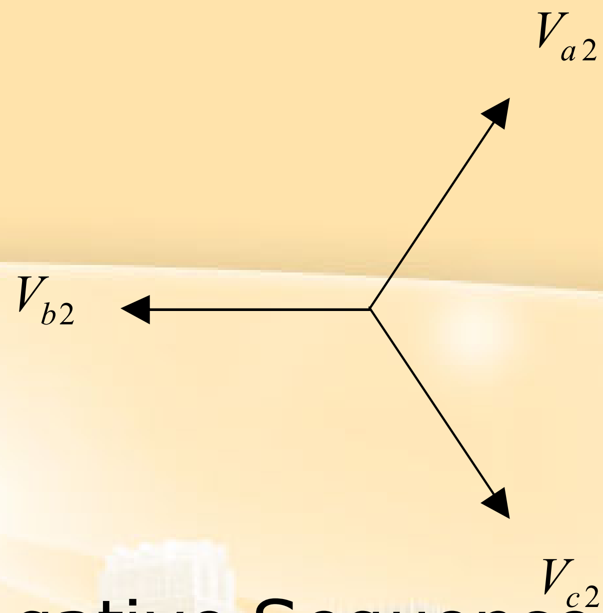
kVA Base for Motors

kVA/hp	hp rating
1.00	Induction < 100 hp
1.00	Synchronous 0.8 pf
0.95	Induction 100 < 999 hp
0.90	Induction > 1000 hp
0.80	Synchronous 1.0 pf

SYMMETRICAL COMPONENTS



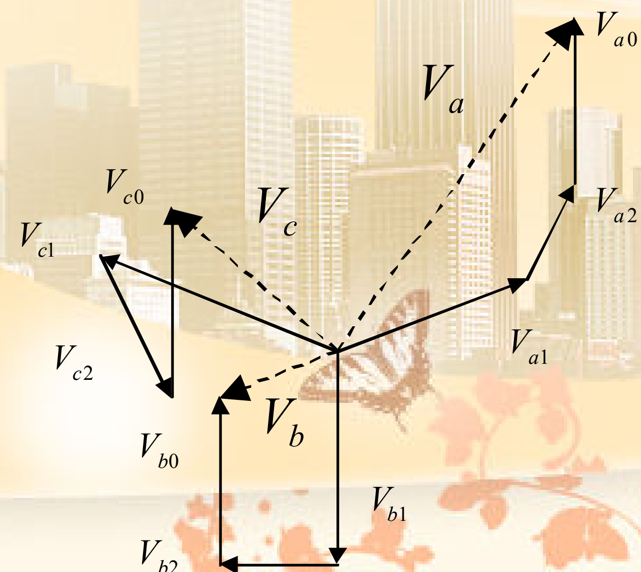
Positive Sequence



Negative Sequence

$$V_{a0} = V_{b0} = V_{c0}$$

Zero Sequence



Unbalanced Phasors

Symmetrical Components of Unbalanced Three-phase Phasor

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right)$$

$$V_{a1} = \frac{1}{3}\left(V_a + a V_b + a^2 V_c\right)$$

$$V_{a2} = \frac{1}{3}\left(V_a + a^2 V_b + a V_c\right)$$

Symmetrical Components of Unbalanced Three-phase Phasor

In matrix form:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Power System Short Circuit Calculations

Sequence Networks

Fault Point

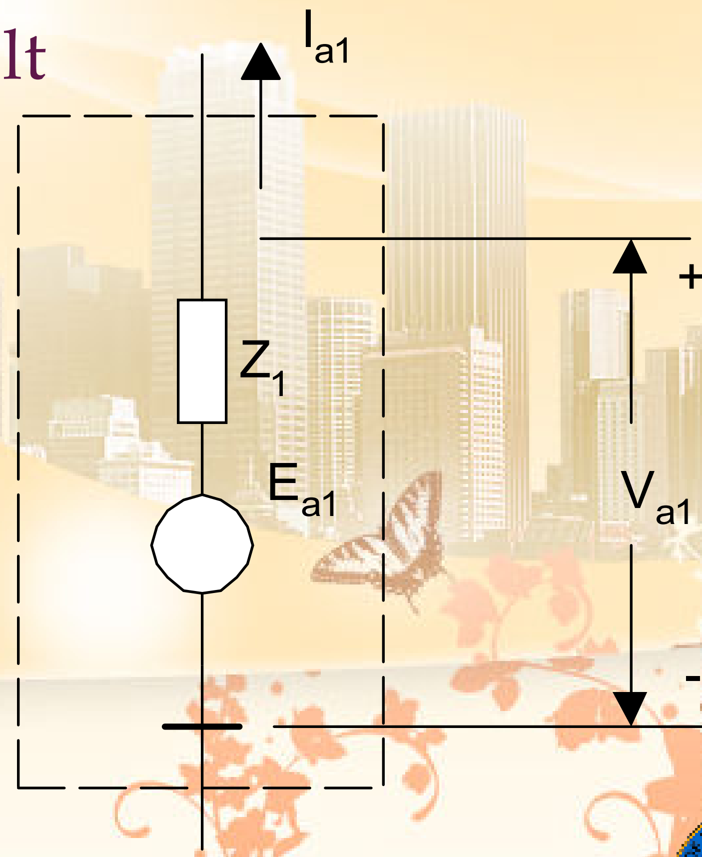
The fault point of a system is that point to which the unbalanced connection is attached to an otherwise balanced system.

Definition of Sequence Networks

Positive-sequence Network

E_{a1} = Thevenin's equivalent voltage as seen at the fault point

Z_1 = Thevenin's equivalent impedance as seen from the fault point



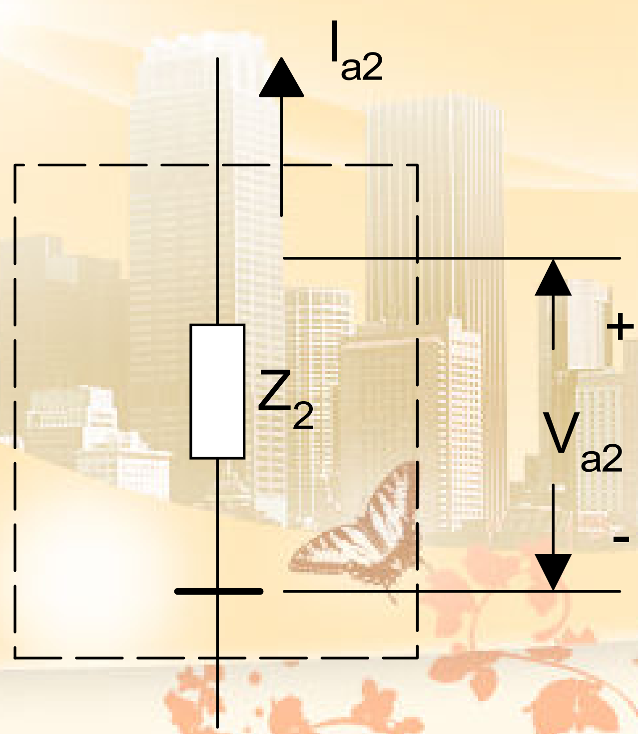
$$V_{a1} = E_{a1} - I_{a1} Z_1$$

Definition of Sequence Networks

Negative-sequence Network

Z_2 = Thevenin's equivalent negative-sequence impedance as seen at the fault point

$$V_{a2} = -I_{a2} Z_2$$

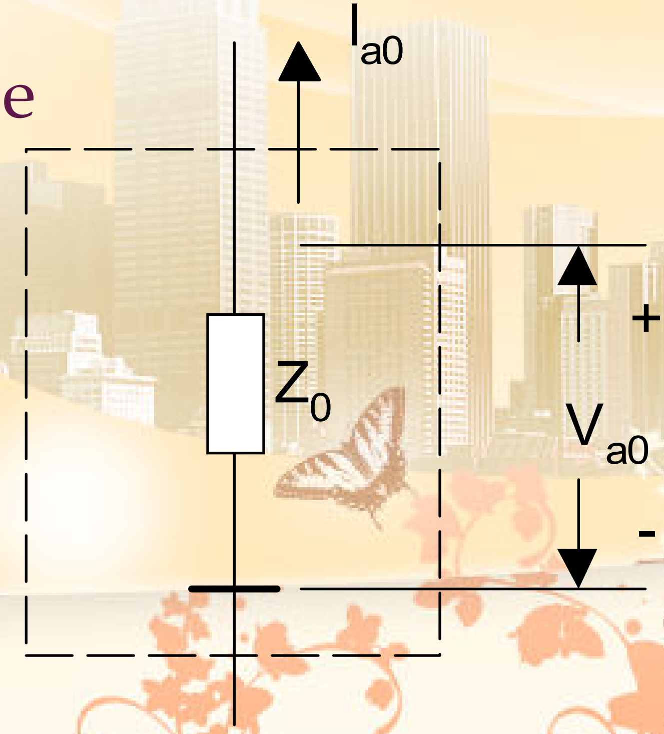


Definition of Sequence Networks

Zero-sequence Network

Z_0 = Thevenin's equivalent zero-sequence impedance as seen at the fault point

$$V_{a0} = -I_{a0} Z_0$$

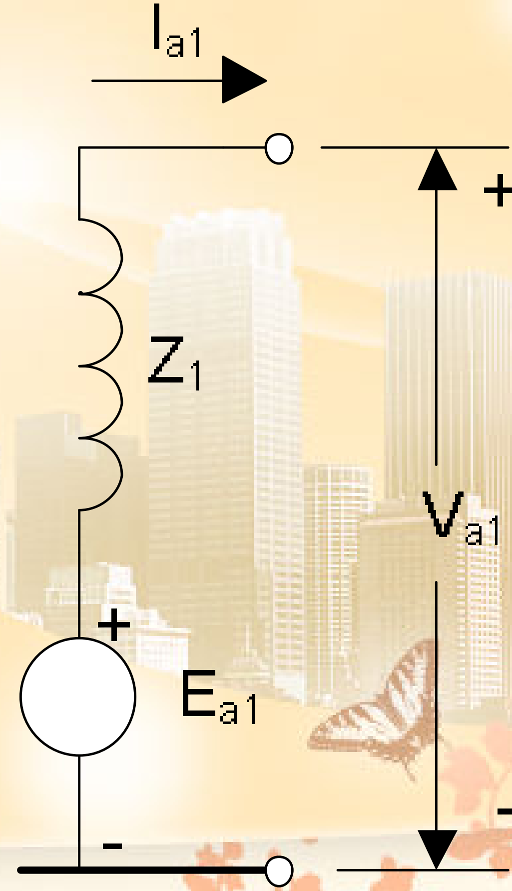


Power System Short Circuit Calculations

Sequence Network Models of Power System Components

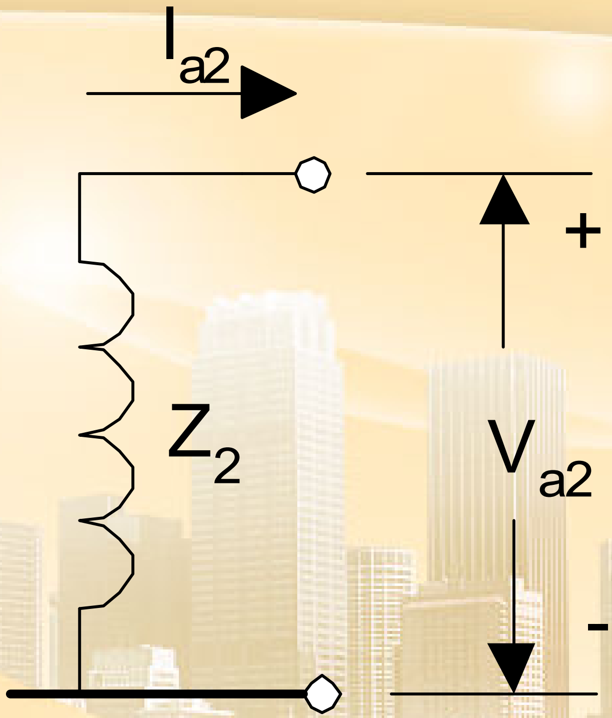
Synchronous Machines (Positive Sequence Network)

$$Z_1 = jx''_d$$



Synchronous Machines (Negative Sequence Network)

$$Z_2 = j\frac{x''_d + x''_q}{2}$$



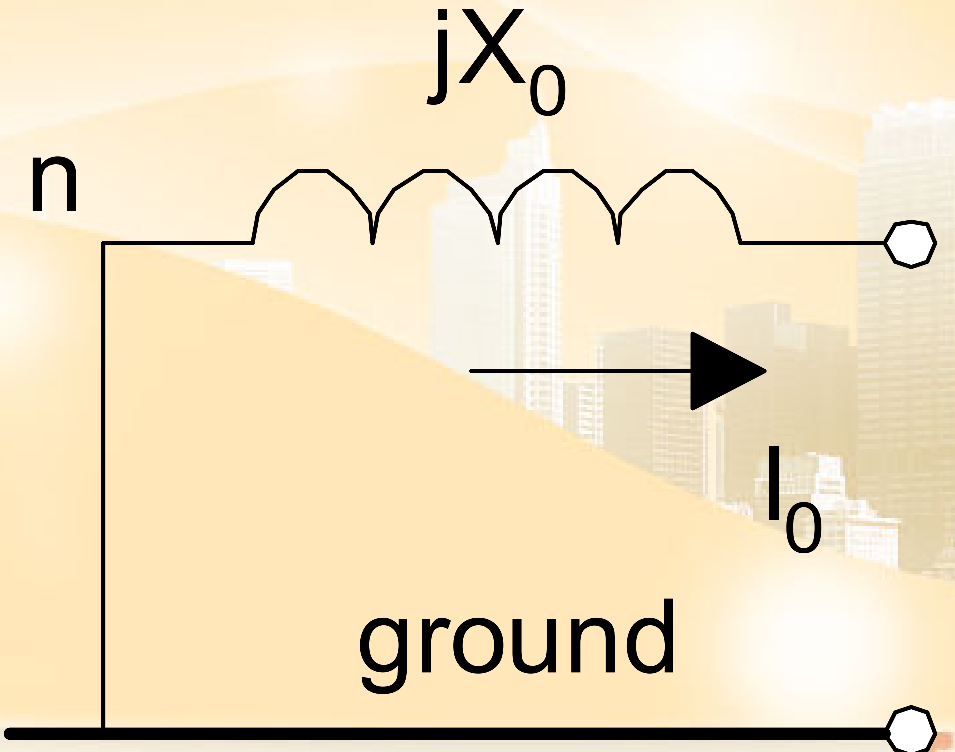
Where:

x''_d = direct-axis sub-transient reactance

x''_q = quadrature-axis sub-transient reactance

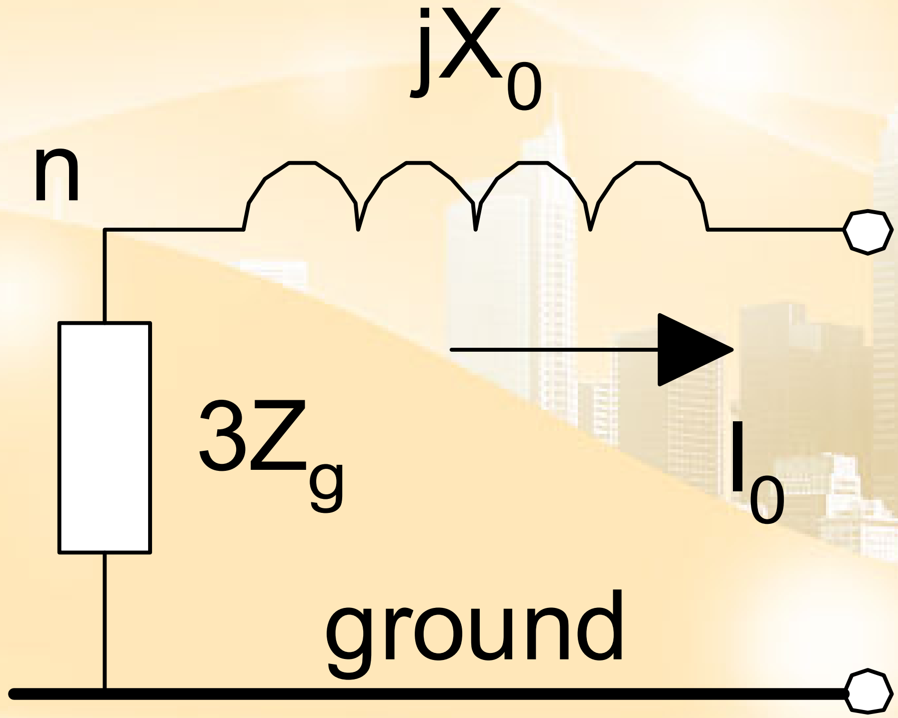
Synchronous Machines (Zero Sequence Network)

Solidly-Grounded Neutral



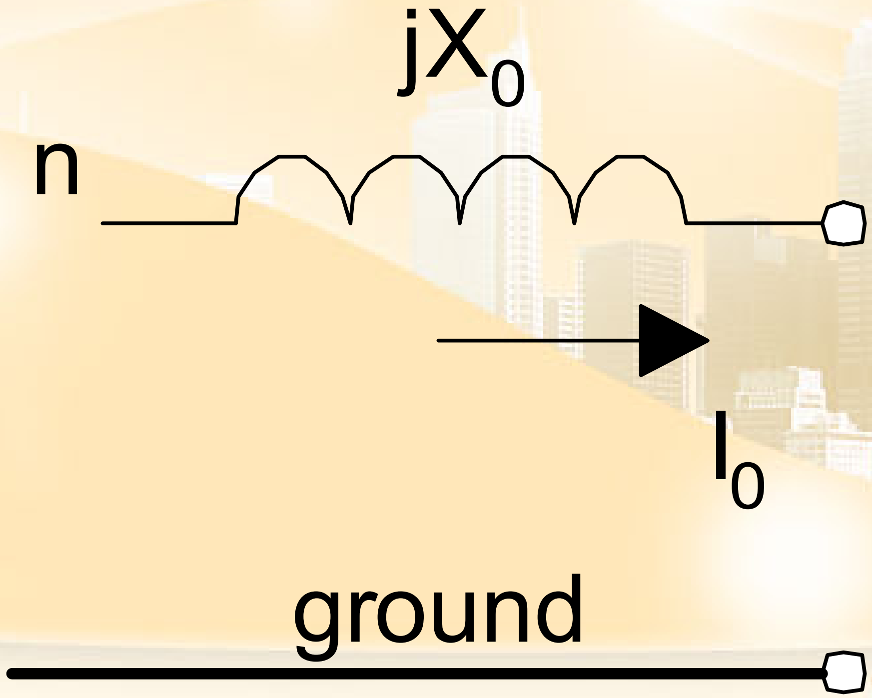
Synchronous Machines (Zero Sequence Network)

Impedance-Grounded Neutral

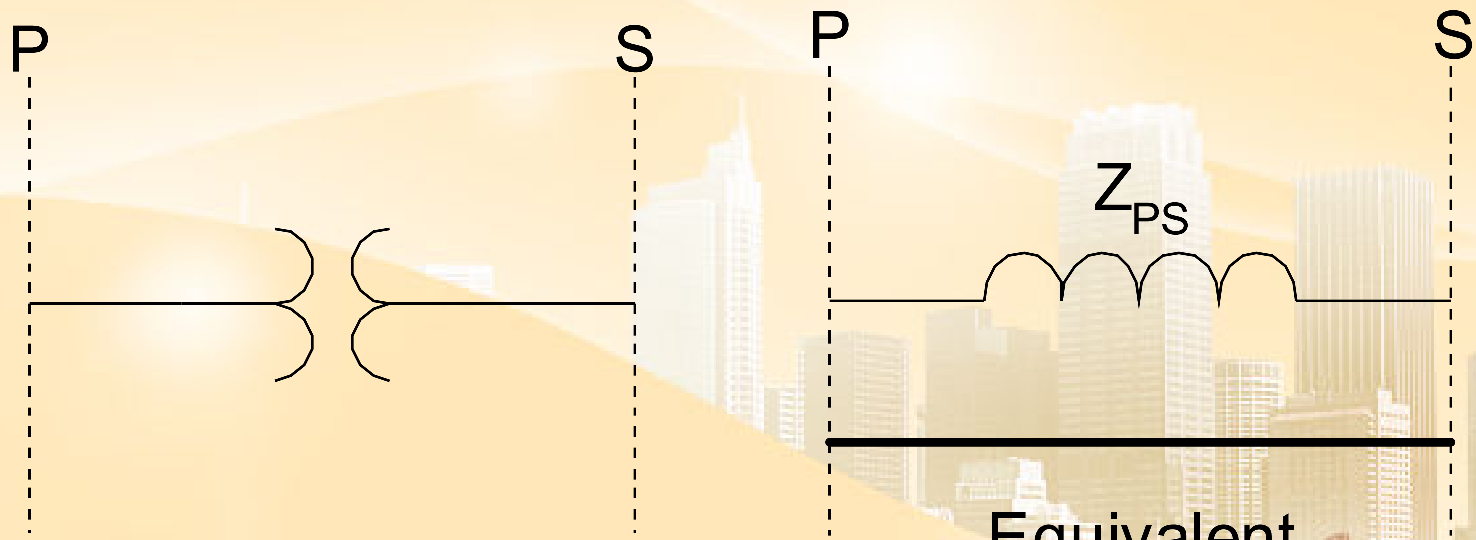


Synchronous Machines (Zero Sequence Network)

Ungrounded-Wye or Delta Connected Generators



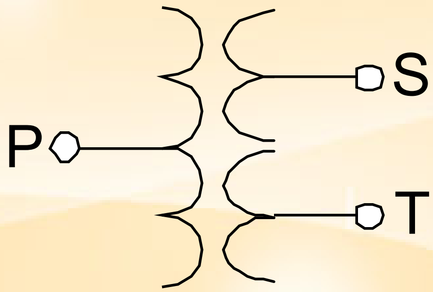
Two-Winding Transformers (Positive Sequence Network)



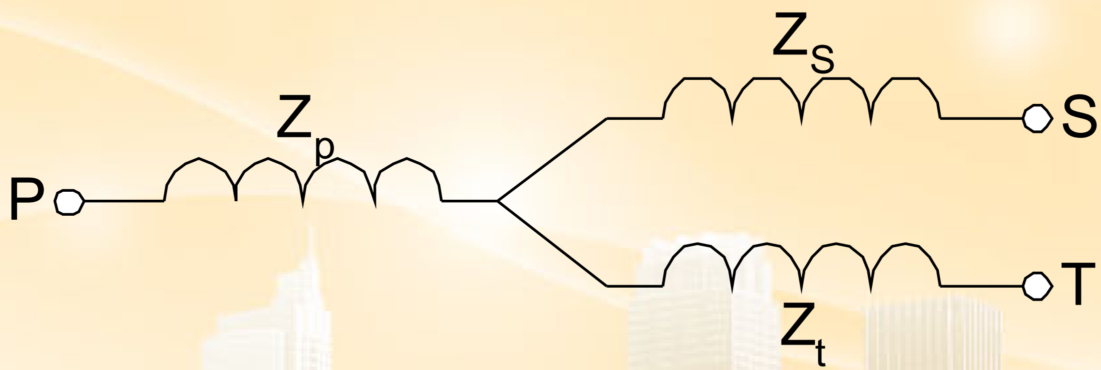
Standard Symbol

Equivalent Positive-seq. Network

Three-Winding Transformers (Positive Sequence Network)



Standard Symbol



Equivalent Positive-Sequence Network

$$Z_{ps} = Z_p + Z_s$$

$$Z_{pt} = Z_p + Z_t$$

$$Z_{st} = Z_s + Z_t$$

$$Z_p = \frac{1}{2}\left(Z_{ps} + Z_{pt} - Z_{st}\right)$$

$$Z_s = \frac{1}{2}\left(Z_{ps} + Z_{st} - Z_{pt}\right)$$

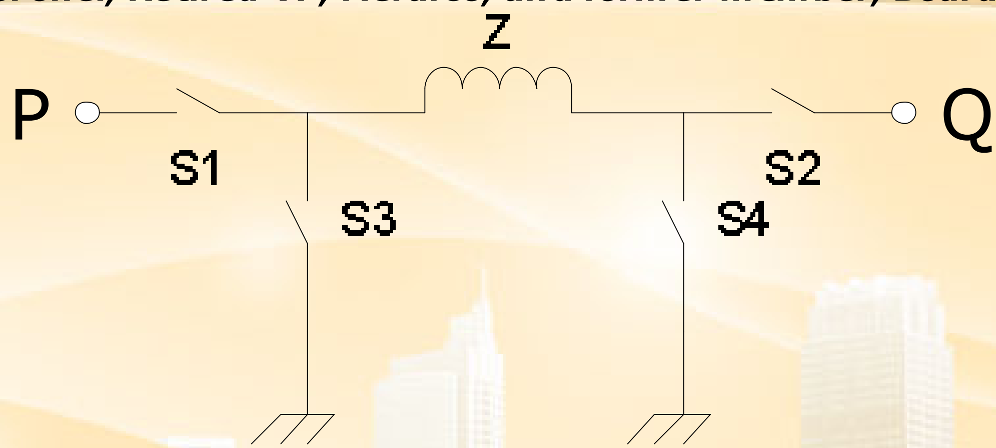
$$Z_t = \frac{1}{2}\left(Z_{pt} + Z_{st} - Z_{ps}\right)$$

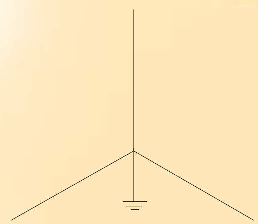
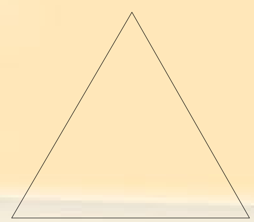
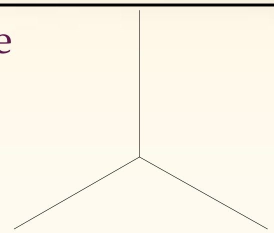
Transformers (Negative Sequence Network)

The negative-sequence network of two-winding and three-winding transformers are modeled in the same way as the positive-sequence network since the positive-sequence and negative-sequence impedances of transformers are equal.

Simplified Derivation of Transformer Zero-Sequence Circuit Modeling

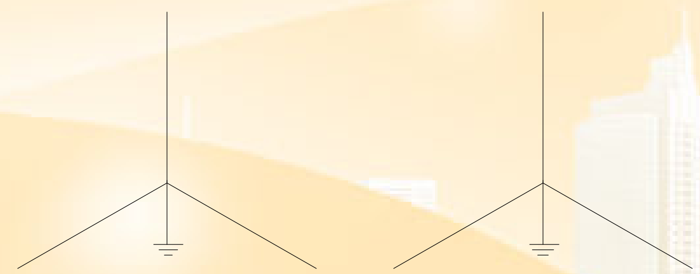
(Thanks to Engr. Antonio C. Coronel, Retired VP, Meralco, and former member, Board of Electrical Engineering)



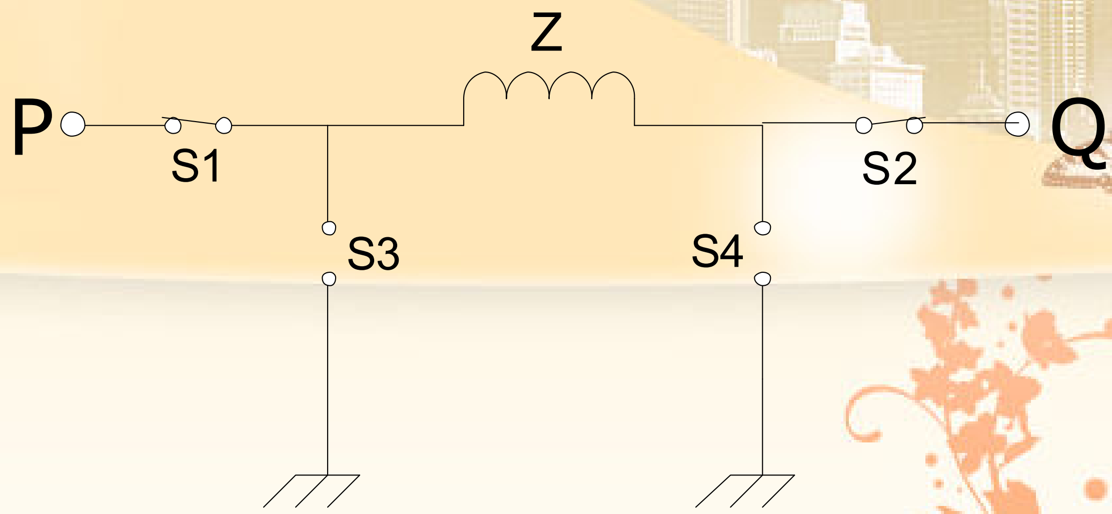
Grounded wye		**S1 = 1 and S3 = 0** **or** **S2 = 1 or S4 = 0**
Delta		**S1 = 0 and S3 = 1** **or** **S2 = 0 and S4 = 1**
Ungrounded wye		**S1 = 0 and S3 = 0** **or** **S2 = 0 and S4 = 0**

Simplified Derivation of Transformer Zero-Sequence Circuit Modeling

Grounded wye – Grounded wye

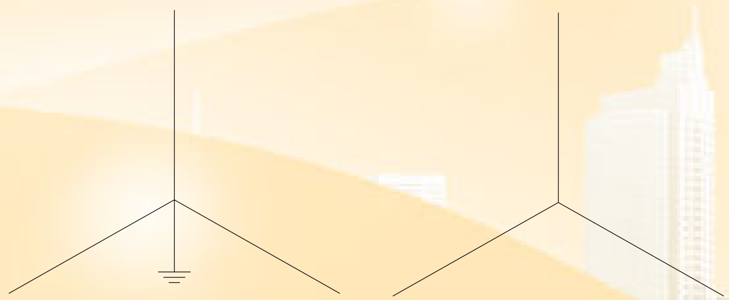


S1 = 1
S2 = 1
S3 = 0
S4 = 0

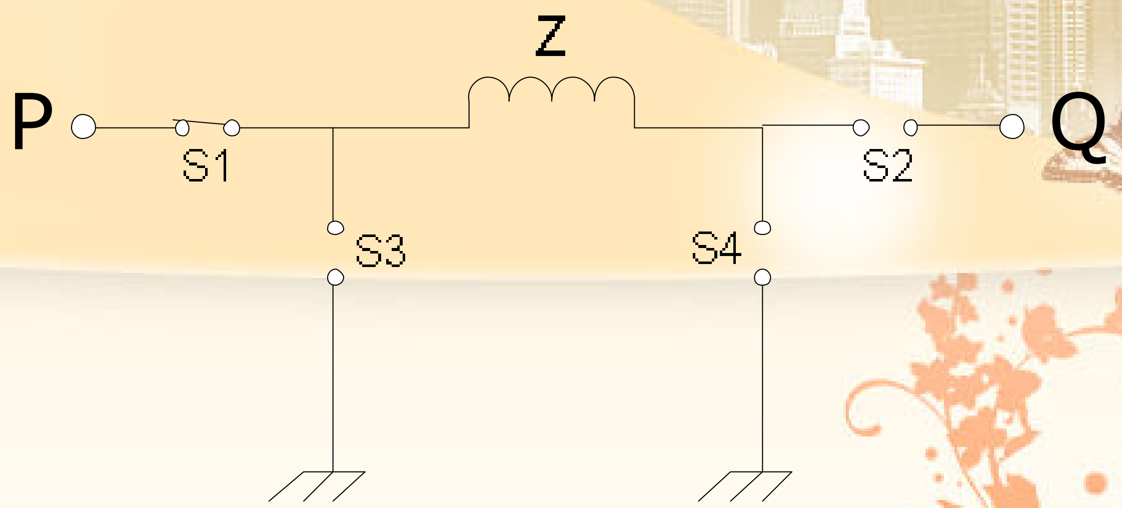


Simplified Derivation of Transformer Zero-Sequence Circuit Modeling

Grounded wye – Ungrounded wye

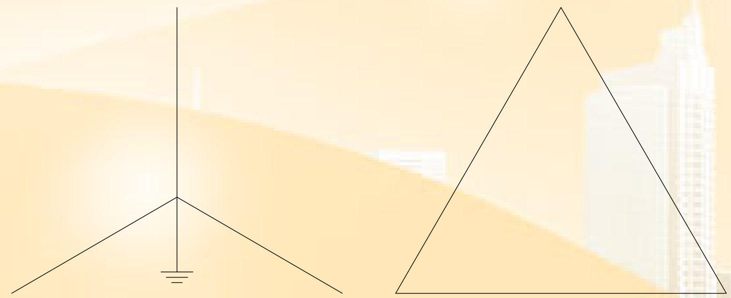


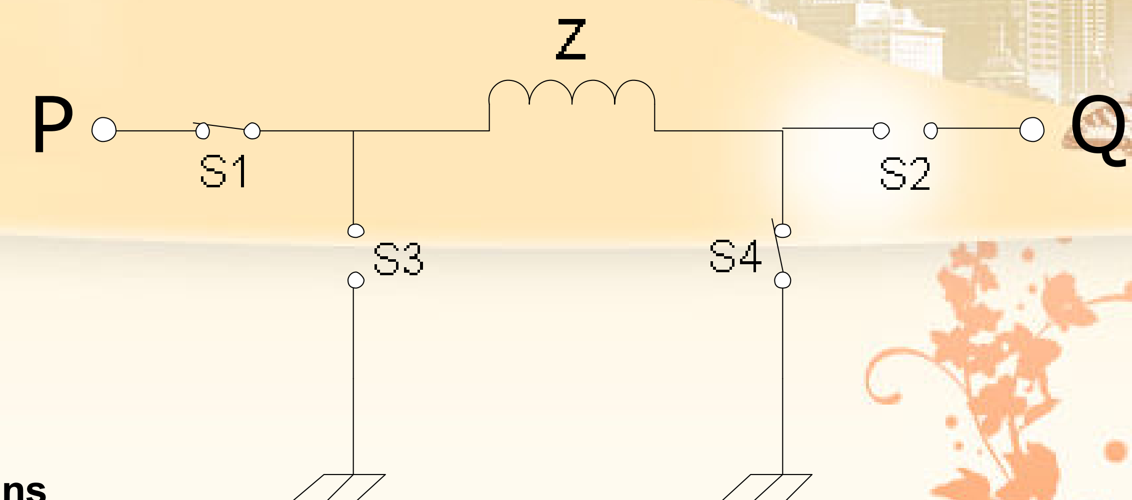
S1 = 1
S2 = 0
S3 = 0
S4 = 0



Simplified Derivation of Transformer Zero-Sequence Circuit Modeling

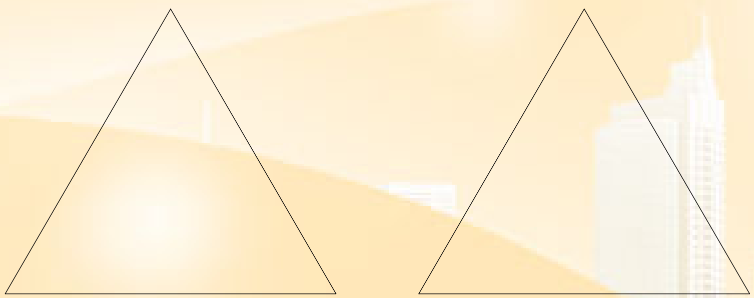
Grounded wye – Delta

	**S1 = 1** **S2 = 0** **S3 = 0** **S4 = 1**

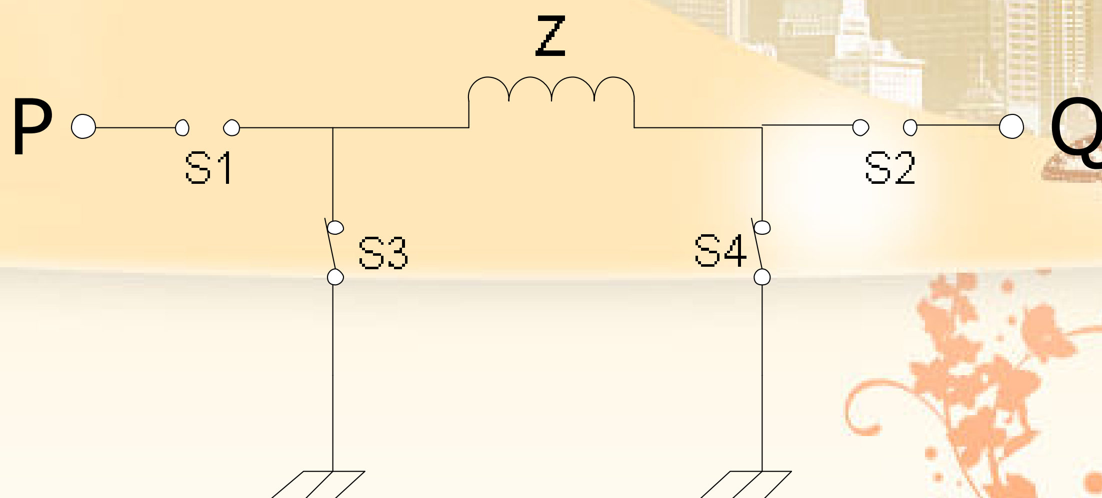


Simplified Derivation of Transformer Zero-Sequence Circuit Modeling

Delta – Delta

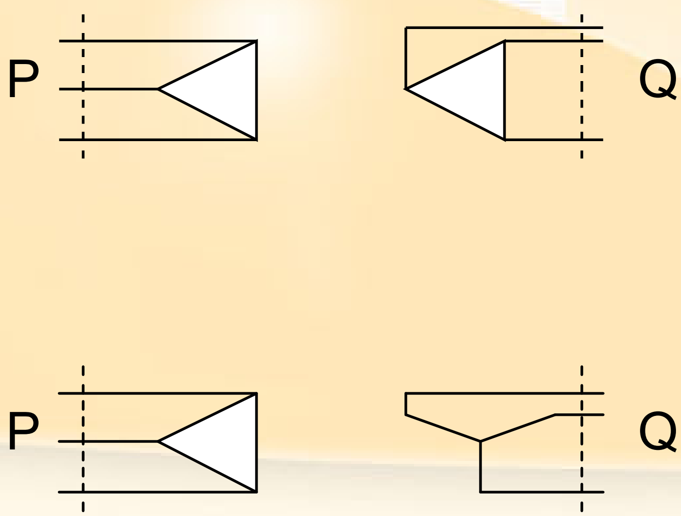


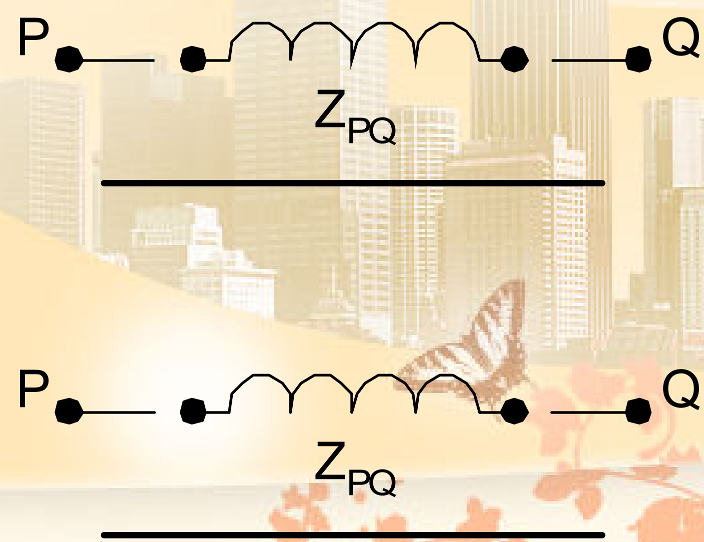
S1 = 0
S2 = 0
S3 = 0
S4 = 0



Transformers (Zero-Sequence Circuit Model)

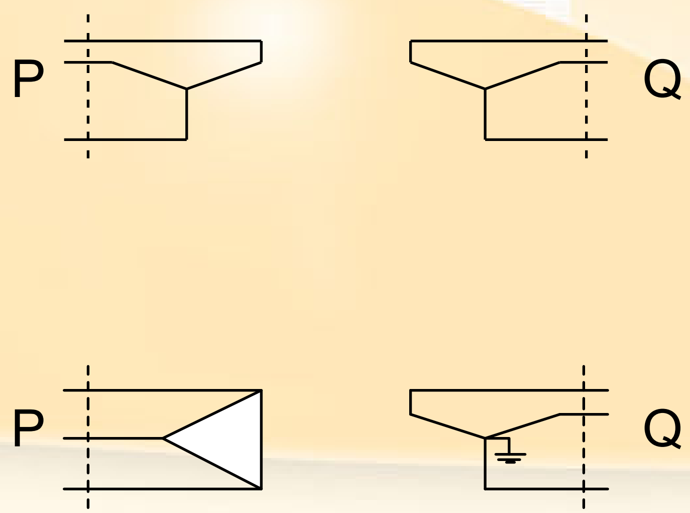
Transformer Connection	Zero-Sequence Circuit Equivalent

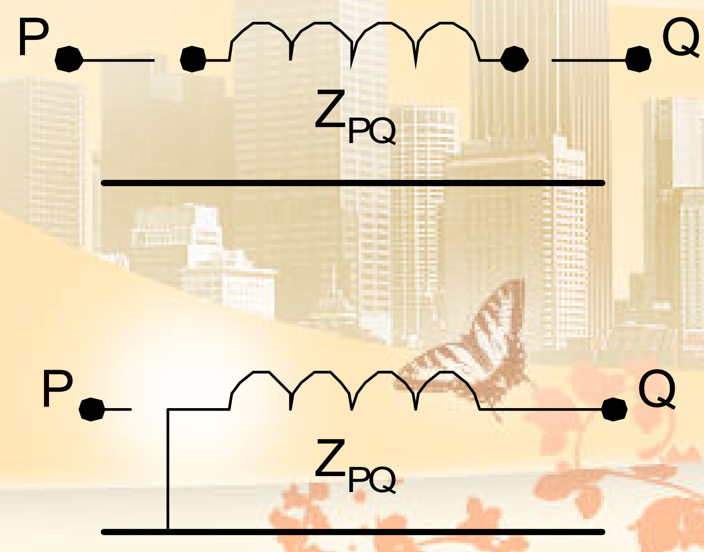




Transformers (Zero-Sequence Circuit Model)

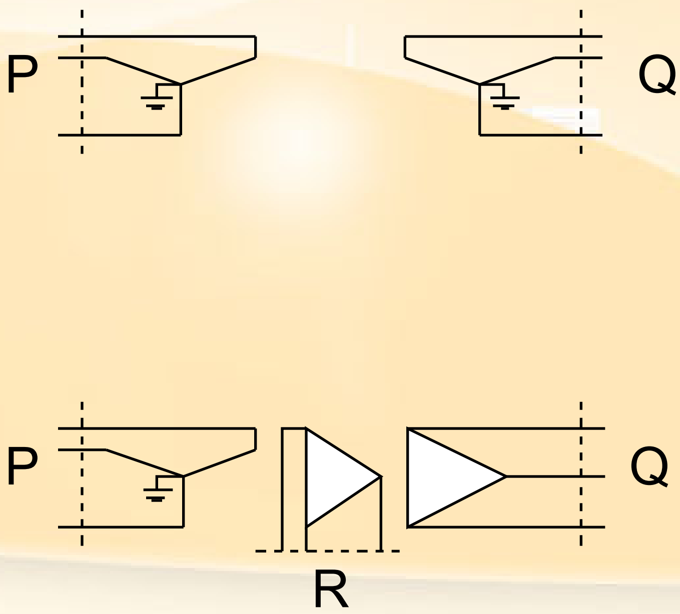
Transformer Connection	Zero-Sequence Circuit Equivalent

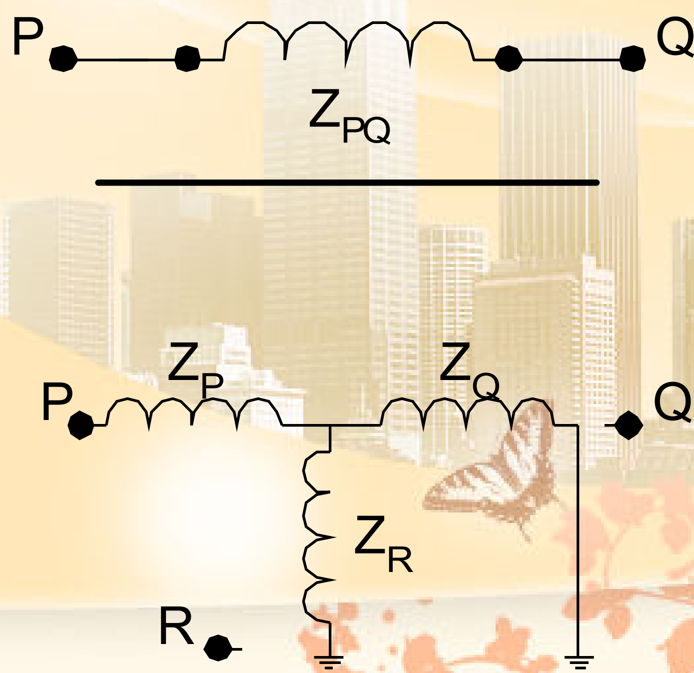




Transformers (Zero-Sequence Circuit Model)

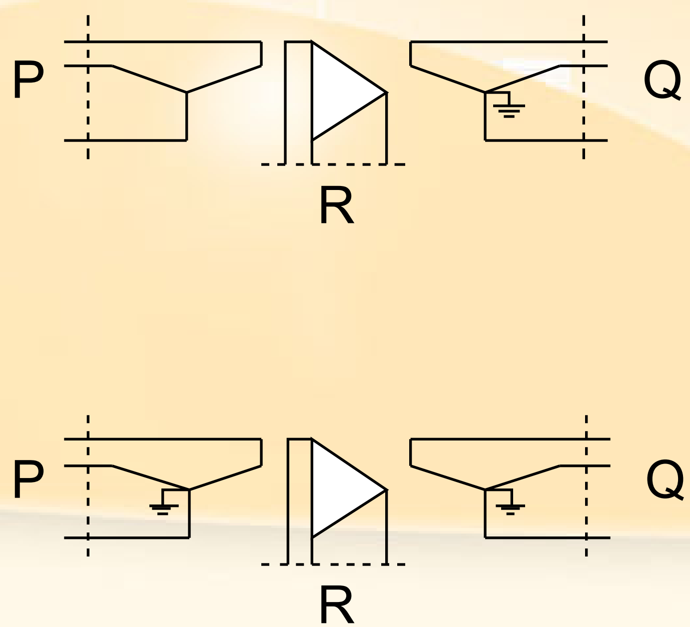
Transformer Connection	Zero-Sequence Circuit Equivalent



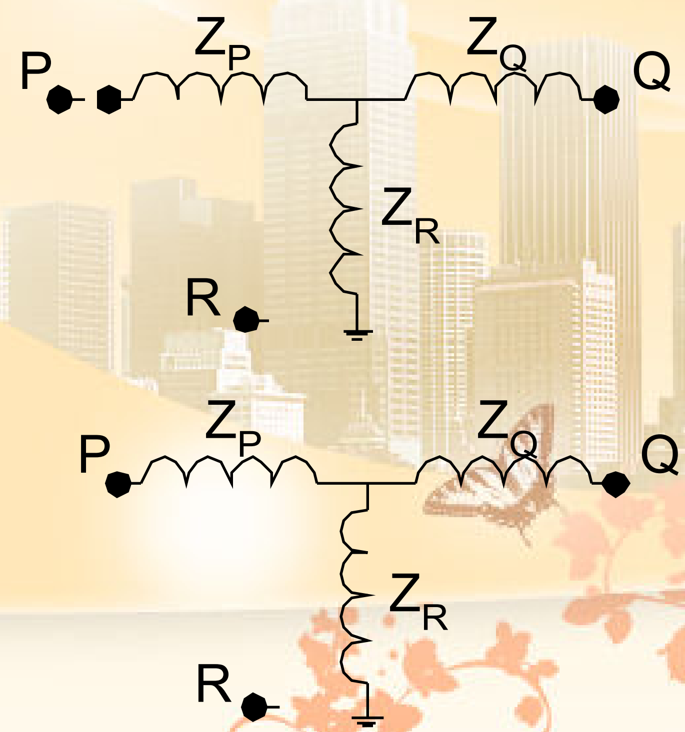


Transformers (Zero-Sequence Circuit Model)

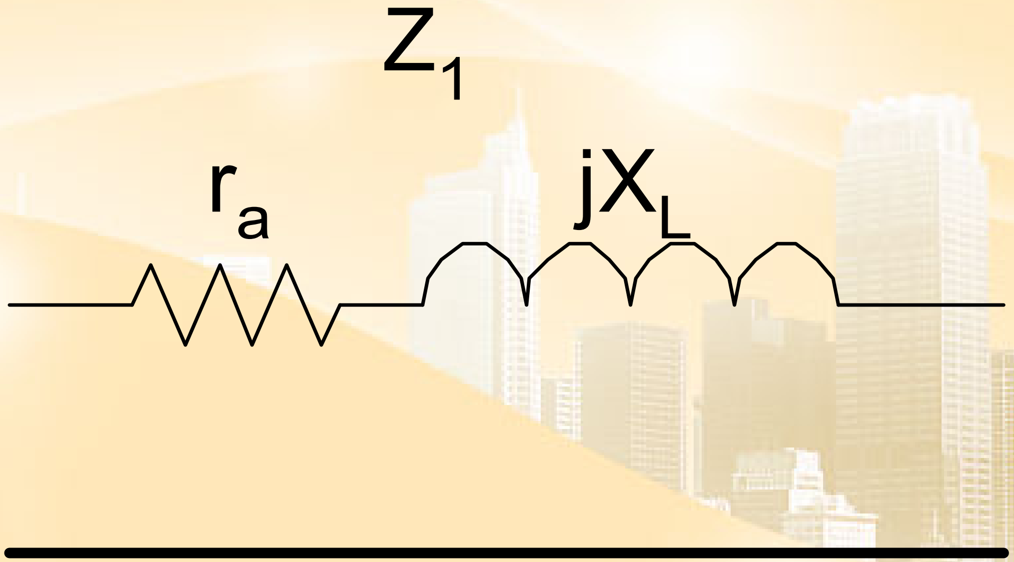
Transformer Connection



Zero-Sequence Circuit Equivalent



Transmission Lines (Positive Sequence Network)



Transmission Lines (Negative Sequence Network)

The same model as the positive-sequence network is used for transmission lines inasmuch as the positive-sequence and negative-sequence impedances of transmission lines are the same

Transmission Lines (Zero Sequence Network)

The zero-sequence network model for a transmission line is the same as that of the positive- and negative-sequence networks. The sequence impedance of the model is of course the zero-sequence impedance of the line. This is normally higher than the positive- and negative-sequence impedances because of the influence of the earth's resistivity and the ground wire/s.

Power System Short Circuit Calculations

Classification of Power System Short Circuits

Shunt Faults

- Single line-to-ground faults
- Double line-to-ground faults
- Line-to-line faults
- Three-phase faults

Series Faults

- One-line open faults
- Two-line open faults

Combination of Shunt and Series Faults

- Single line-to-ground and one-line open
- Double line-to-ground and one-line open faults
- Line-to-line and one-line open faults
- Three-phase and one-line open faults

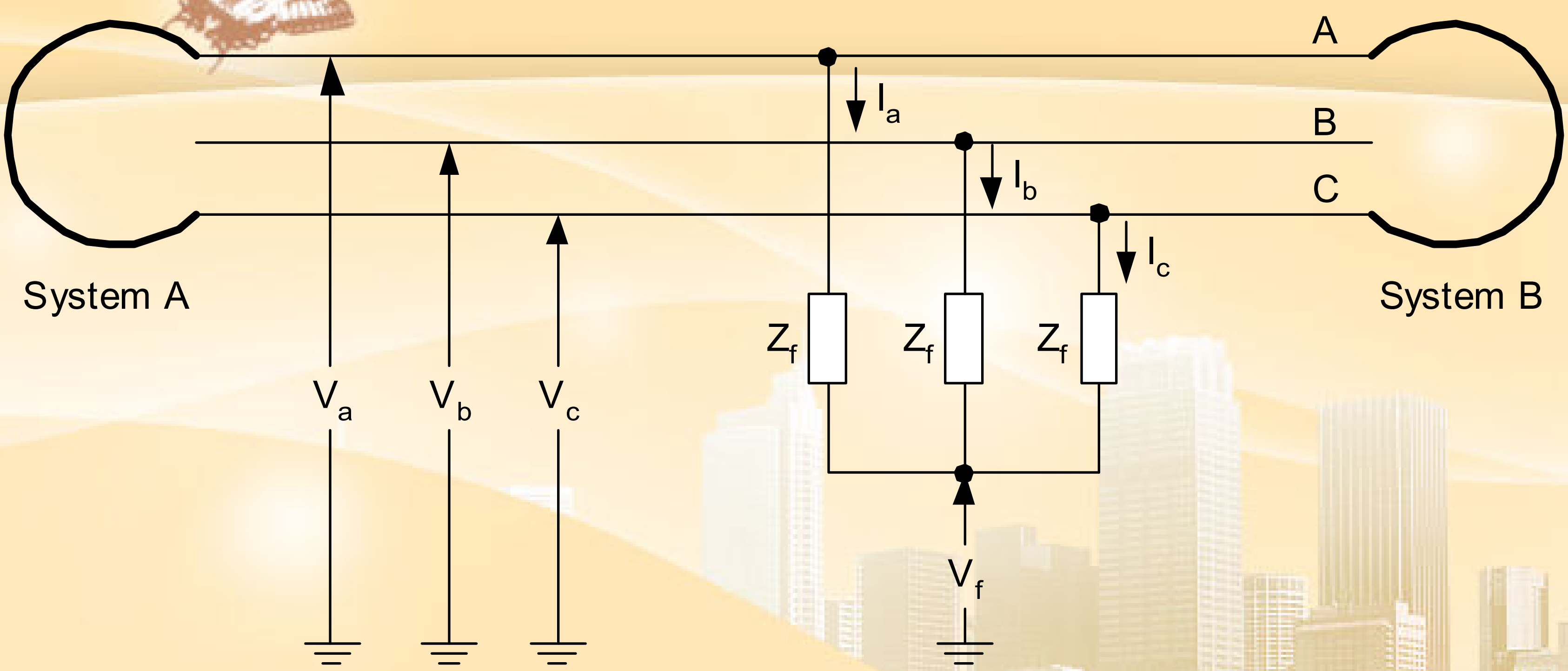
Combination of Shunt and Series Faults

- Single line-to-ground and two-line open faults
- Double line-to-ground and two-line open faults
- Line-to-line and two-line open faults
- Three-phase and two-line open faults

Balanced Faults

Symmetrical or
Three-Phase Faults

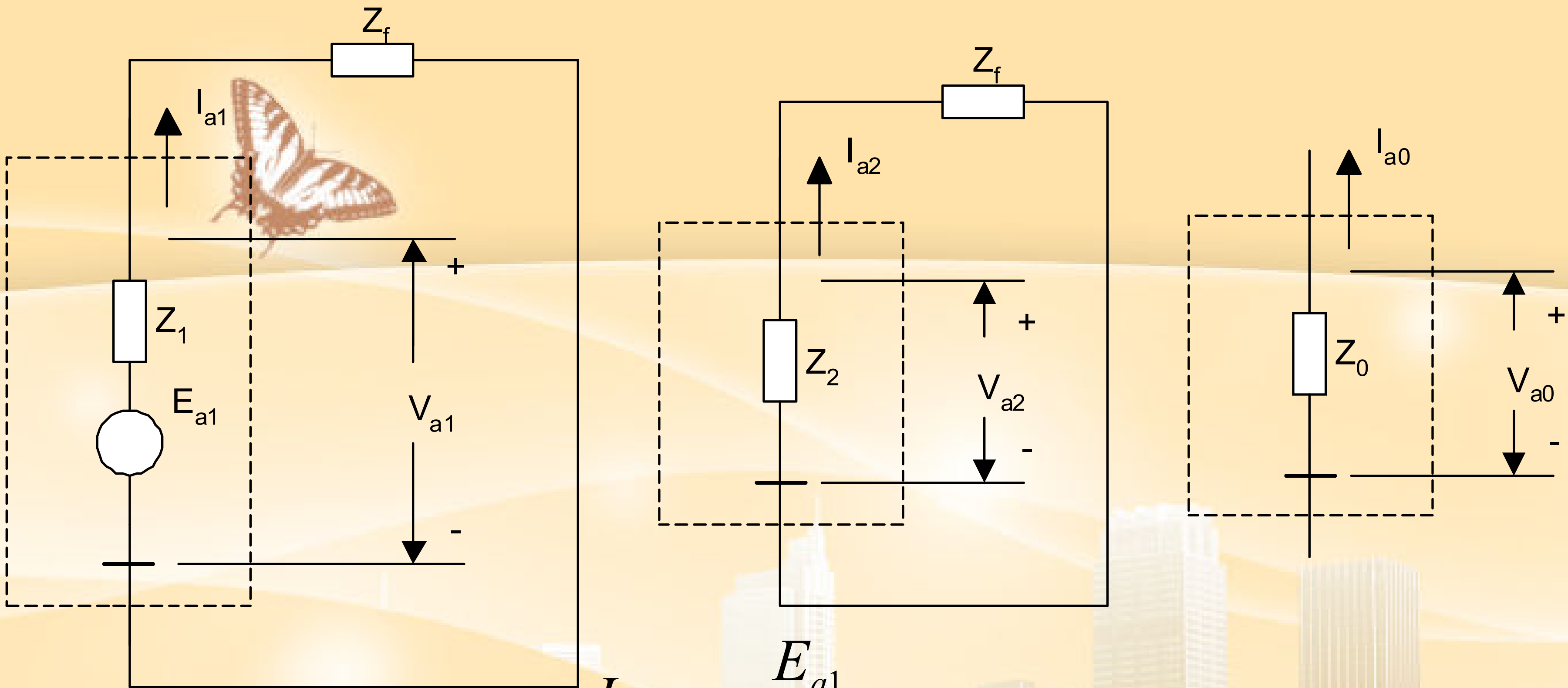
Derivation of Sequence Network Interconnections



Boundary conditions:

$$I_a + I_b + I_c = 0 \qquad \text{Eq'n (1)}$$

$$V_F = V_a - I_a Z_f = V_b - I_b Z_f = V_c - I_c Z_f \qquad \text{Eq'n (2)}$$



$$I_{a1} = \frac{E_{a1}}{Z_1 + Z_f}$$

$$I_f = I_{a0} + I_{a1} + I_{a2}$$

$$I_f = I_{a1} = \frac{E_{a1}}{Z_1 + Z_f}$$

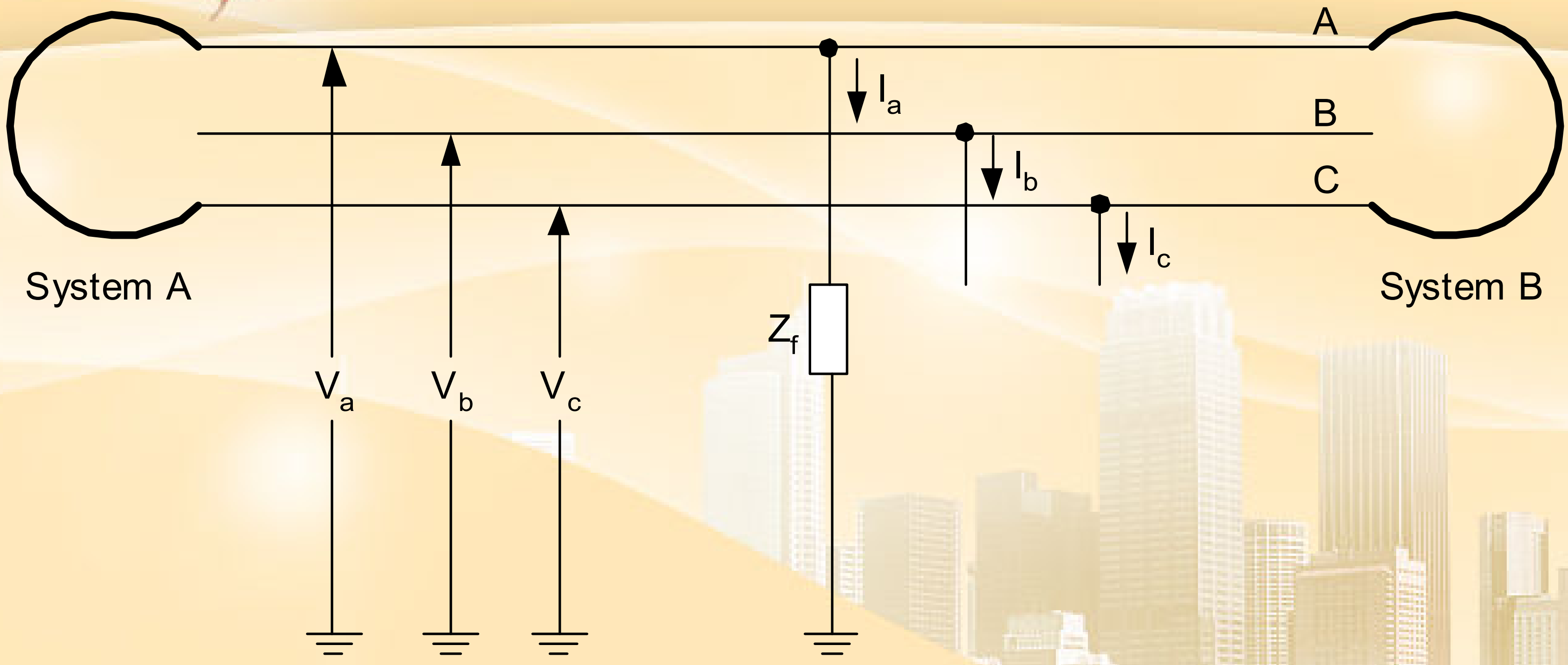
$$Z_f = 0,$$

$$I_f = I_{a1} = \frac{E_{a1}}{Z_1}$$

Unbalanced Faults

Single Line-to-Ground Faults

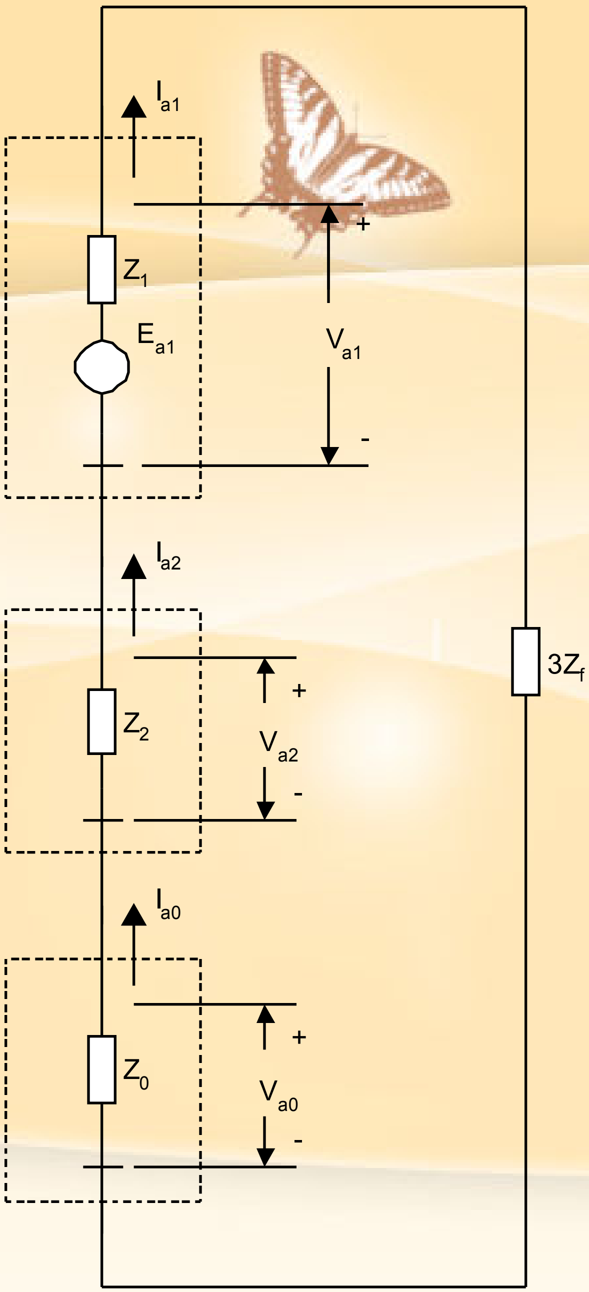
Derivation of Sequence Network Interconnections



Boundary conditions:

$$I_b = I_c = 0$$

$$V_a = I_a Z_f = 0$$



$$I_{a0} = I_{a1} = I_{a2} = \frac{E_{a1}}{Z_0 + Z_1 + Z_2 + 3Z_f}$$

$$If \quad Z_f = 0$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{E_{a1}}{Z_0 + Z_1 + Z_2}$$

$$Z_1 = Z_2$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{E_{a1}}{Z_0 + 2Z_1}$$

$$I_f = I_a = I_{a0} + I_{a1} + I_{a2} = 3I_{a1} = 3I_{a0}$$

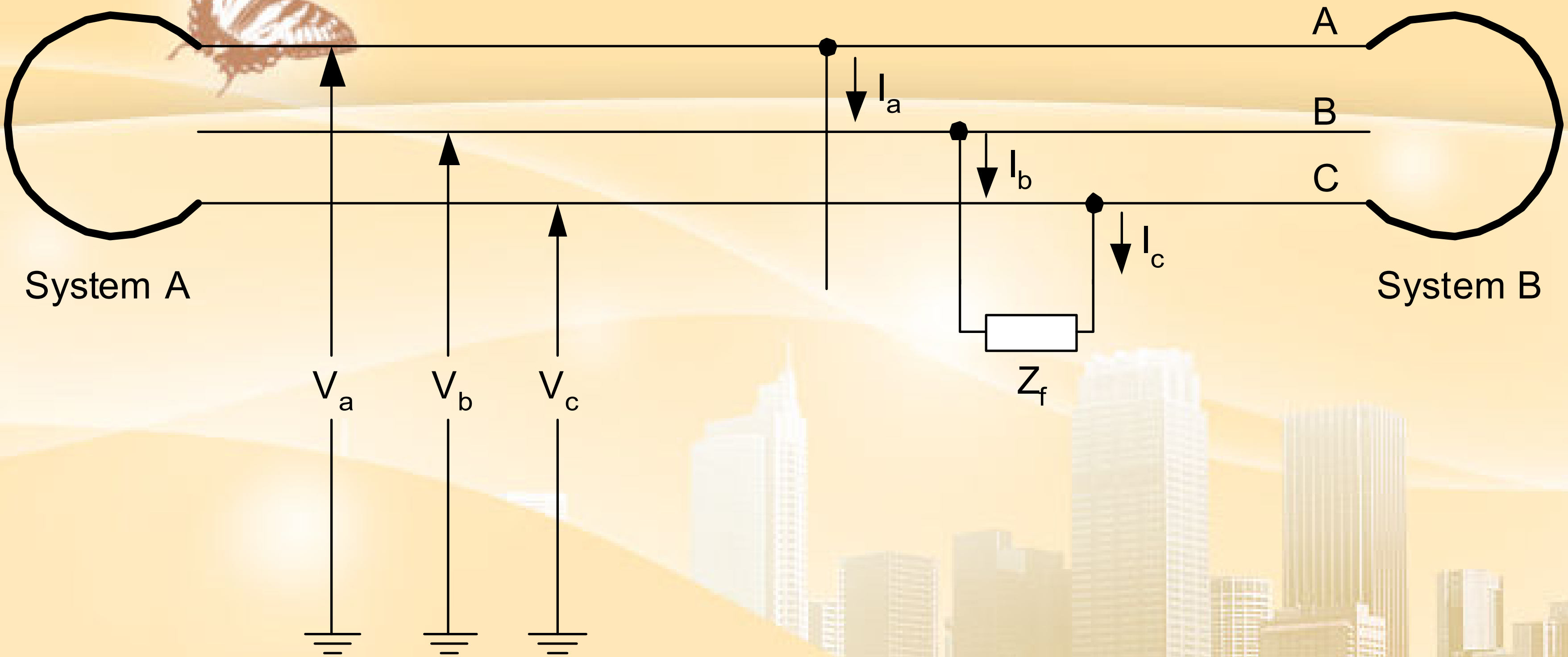
$$If \quad Z_f = 0 \quad and \quad Z_1 = Z_2$$

$$I_f = I_a = \frac{3E_{a1}}{Z_0 + 2Z_1}$$

Unbalanced Faults

Line-to-Line Faults

Derivation of Sequence Network Interconnections

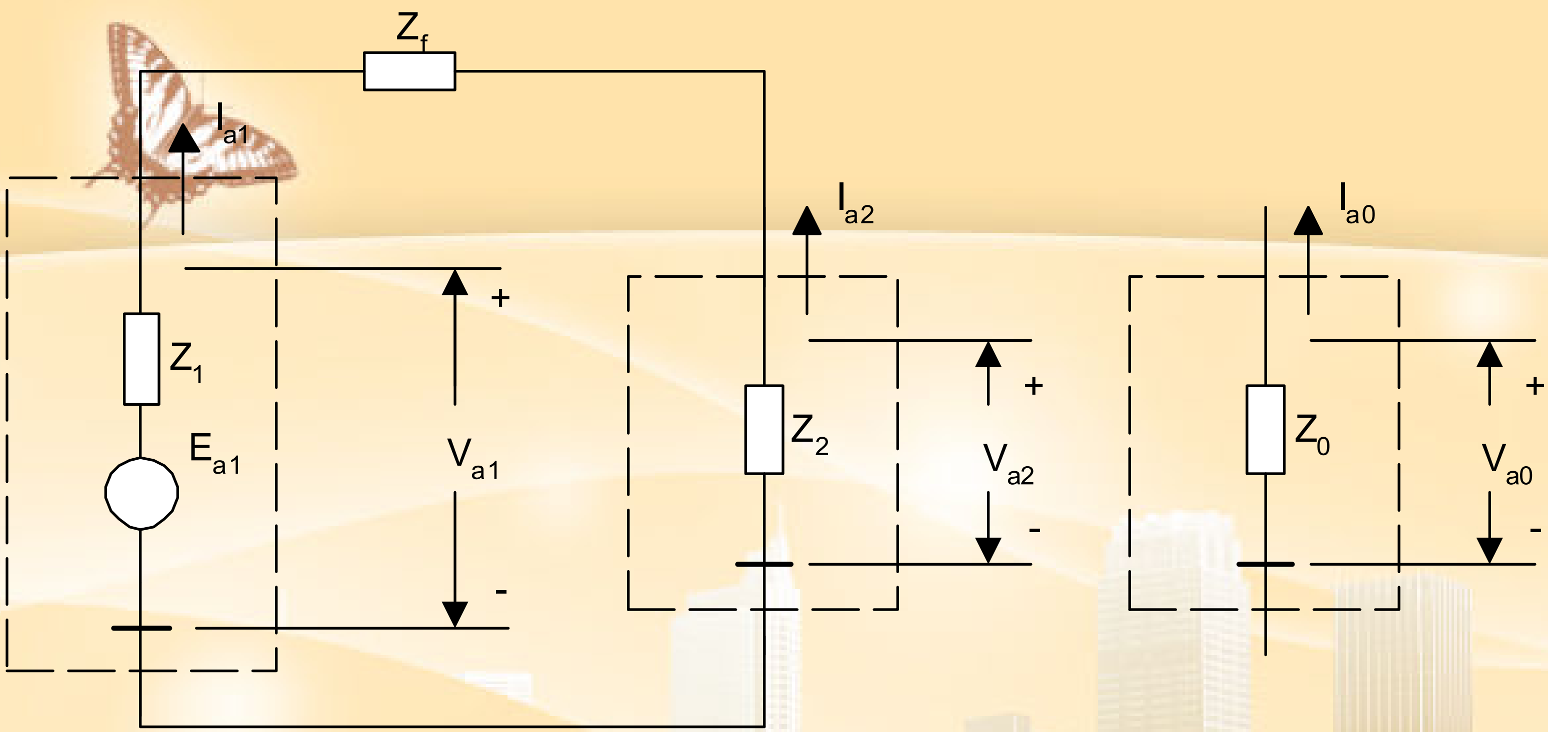


Boundary conditions:

$$I_a = 0$$

$$I_b = -I_c$$

$$V_b - I_b Z_f = V_c;\ \text{or}\ \ V_b - V_c = I_b Z_f$$



$$I_{a1} = \frac{E_{a1}}{Z_1 + Z_2 + Z_f}$$

$$If\ Z_f = 0\ and\ Z_1 = Z_2$$

$$I_{a1} = \frac{E_{a1}}{2Z_1}$$

The fault current

$$I_f = I_b = -I_c = I_{a0} + a^2 I_{a1} + aI_{a2}$$

$$I_{ao} = 0; \quad I_{a1} = -I_{a2}$$

$$I_f = \left(a^2 - a\right) I_{a1} = -j\sqrt{3} I_{a1}$$

thus, with $Z_1 = Z_2$

$$I_f = -j\sqrt{3}\left[\frac{E_{a1}}{2Z_1 + Z_f}\right]$$

if $Z_f = 0$

$$\left|I_f\right| = \left|-j\frac{\sqrt{3}E_{a1}}{2Z_1}\right| = \left(\frac{\sqrt{3}}{2}\right)\left|\frac{E_{a1}}{Z_1}\right|$$

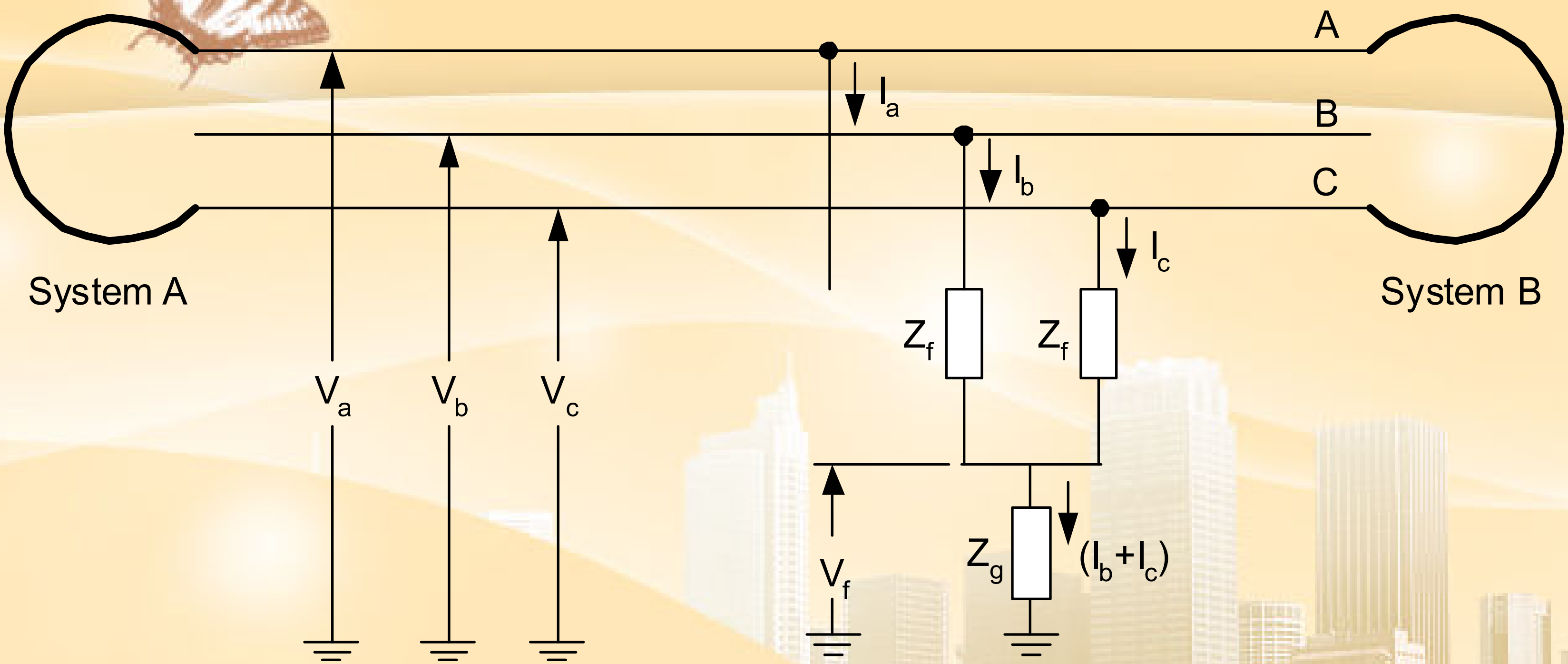
$$I_{f[3\phi]} = \frac{E_{a1}}{Z_1}$$

$$I_{f[L-L]} = \frac{\sqrt{3}}{2} I_{f[3\phi]}$$

Unbalanced Faults

Double-to-line Ground Fault

Derivation of Sequence Network Interconnections

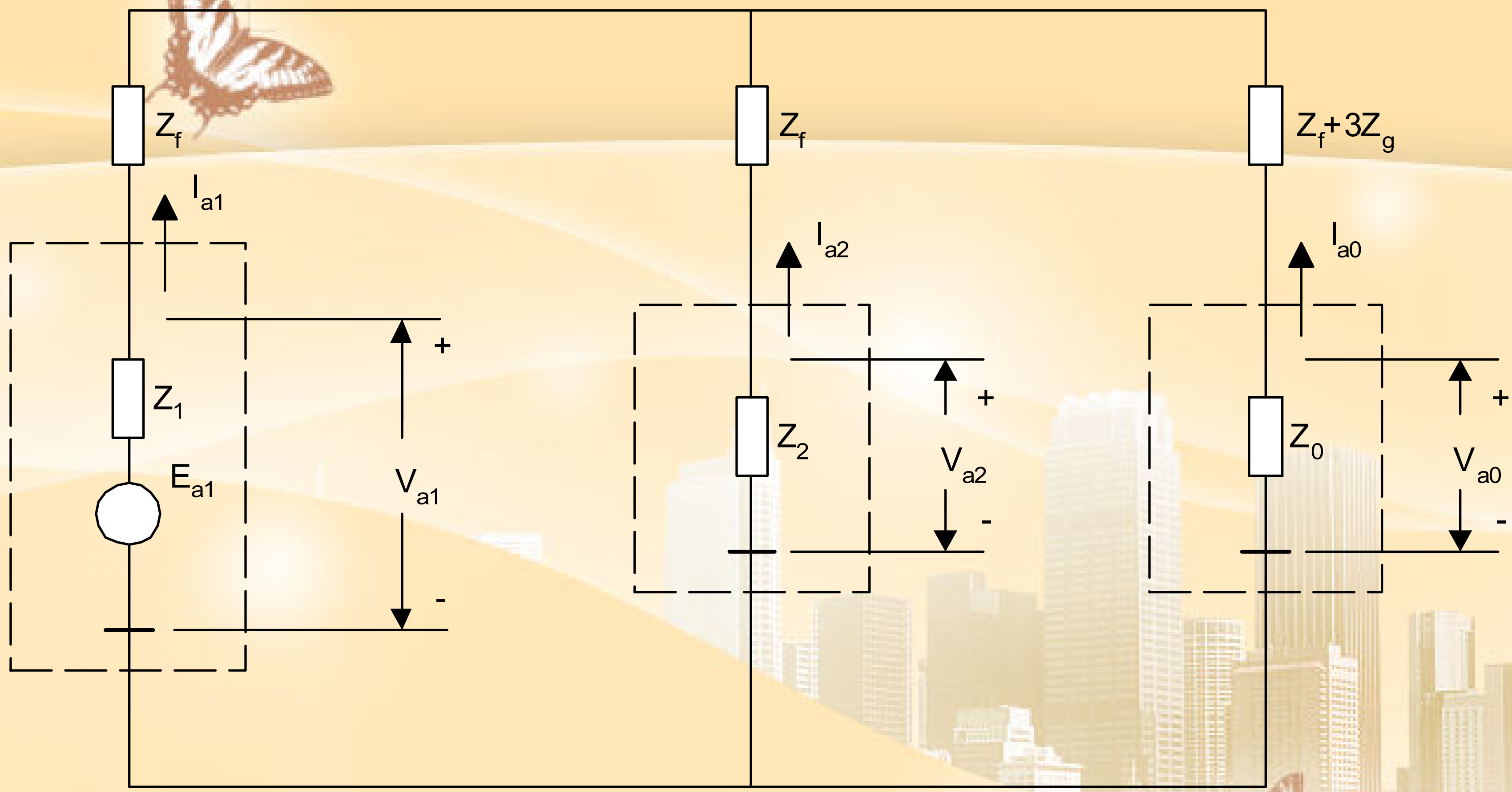


Boundary conditions:

$$I_a = 0 \qquad \text{Eq'n BC-1}$$

$$V_b = I_b Z_f + (I_b + I_c) Z_g \qquad \text{Eq'n BC-2}$$

$$V_c = I_c Z_f + (I_b + I_c) Z_g \qquad \text{Eq'n BC-3}$$



$$I_{a1} = \frac{E_{a1}}{Z_1 + Z_f + \dfrac{(Z_2 + Z_f)(Z_0 + Z_f + 3Z_g)}{Z_2 + Z_0 + 2Z_f + 3Z_g}}$$

Negative-sequence Component:

$$I_{a2} = -I_{a1}\left(\frac{Z_0 + Z_f + 3Z_g}{Z_2 + Z_0 + 2Z_f + 3Z_g}\right)$$

Zero-sequence Component:

$$I_{a0} = -I_{a1}\left(\frac{Z_2 + Z_f}{Z_2 + Z_0 + 2Z_f + 3Z_g}\right)$$

The fault current

$$I_f = I_b + I_c = \left(I_{a0} + a^2 I_{a1} + aI_{a2}\right) + \left(I_{a0} + aI_{a1} + a^2 I_{a2}\right)$$

$$I_f = 2I_{a0} + \left(a^2 + a\right)I_{a1} + \left(a + a^2\right)I_{a2}$$

$$I_f = 2I_{a0} + (-1)I_{a1} + (-1)I_{a2} = 2I_{a0} - \left(I_{a1} + I_{a2}\right)$$

but $I_{a0} + I_{a1} + I_{a2} = 0;$ or $I_{a0} = -\left(I_{a1} + I_{a2}\right)$

thus, $I_f = 3I_{a0}$

$$If\ \ Z_f = Z_g = 0\ \ and\ \ Z_1 = Z_2$$

$$I_{a1} = \frac{E_{a1}}{Z_1 + \dfrac{Z_1 Z_0}{Z_1 + Z_0}} = \frac{(Z_1 + Z_0)E_{a1}}{Z_1^2 + 2Z_1 Z_0}$$

$$I_{a2} = -I_{a1}\left(\frac{Z_0}{Z_1 + Z_0}\right) = -\left(\frac{E_{a1}}{Z_1 + \dfrac{Z_1 Z_0}{Z_1 + Z_0}}\right)\left(\frac{Z_0}{Z_1 + Z_0}\right) =$$

$$-\frac{Z_0 E_{a1}}{Z_1^2 + 2Z_1 Z_0}$$

$$If \quad Z_f = Z_g = 0 \quad and \quad Z_1 = Z_2$$

$$I_{a1} = \frac{E_{a1}}{Z_1 + \dfrac{Z_1 Z_0}{Z_1 + Z_0}} = \frac{(Z_1 + Z_0)E_{a1}}{Z_1^2 + 2Z_1 Z_0}$$

$$I_{a2} = -I_{a1}\left(\frac{Z_0}{Z_1 + Z_0}\right) = -\left(\frac{E_{a1}}{Z_1 + \dfrac{Z_1 Z_0}{Z_1 + Z_0}}\right)\left(\frac{Z_0}{Z_1 + Z_0}\right) =$$

$$-\frac{Z_0 E_{a1}}{Z_1^2 + 2Z_1 Z_0}$$

$$I_{a0} = -I_{a1}\left(\frac{Z_1}{Z_1 + Z_0}\right) = \left(\frac{E_{a1}}{Z_1 + \dfrac{Z_1 Z_0}{Z_1 + Z_0}}\right)\left(\frac{Z_1}{Z_1 + Z_0}\right)$$

$$= \frac{Z_1 E_{a1}}{Z_1^2 + 2Z_1 Z_0} = \frac{E_{a1}}{Z_1 + 2Z_0}$$

$$I_f = 3I_{a0} = \frac{3E_{a1}}{Z_1 + 2Z_0}$$

Voltage Rise Phenomenon

Single-to-line Ground Fault

Unfaulted Phase B Voltage During Single Line-to-Ground Faults

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$

$$V_b = -\left(\frac{E_{a1}}{2Z_1 + Z_0}\right) Z_0 + a^2\left[E_{a1} - \left(\frac{E_{a1}}{2Z_1 + Z_0}\right) Z_1\right] - a\left(\frac{E_{a1}}{2Z_1 + Z_0}\right) Z_1$$

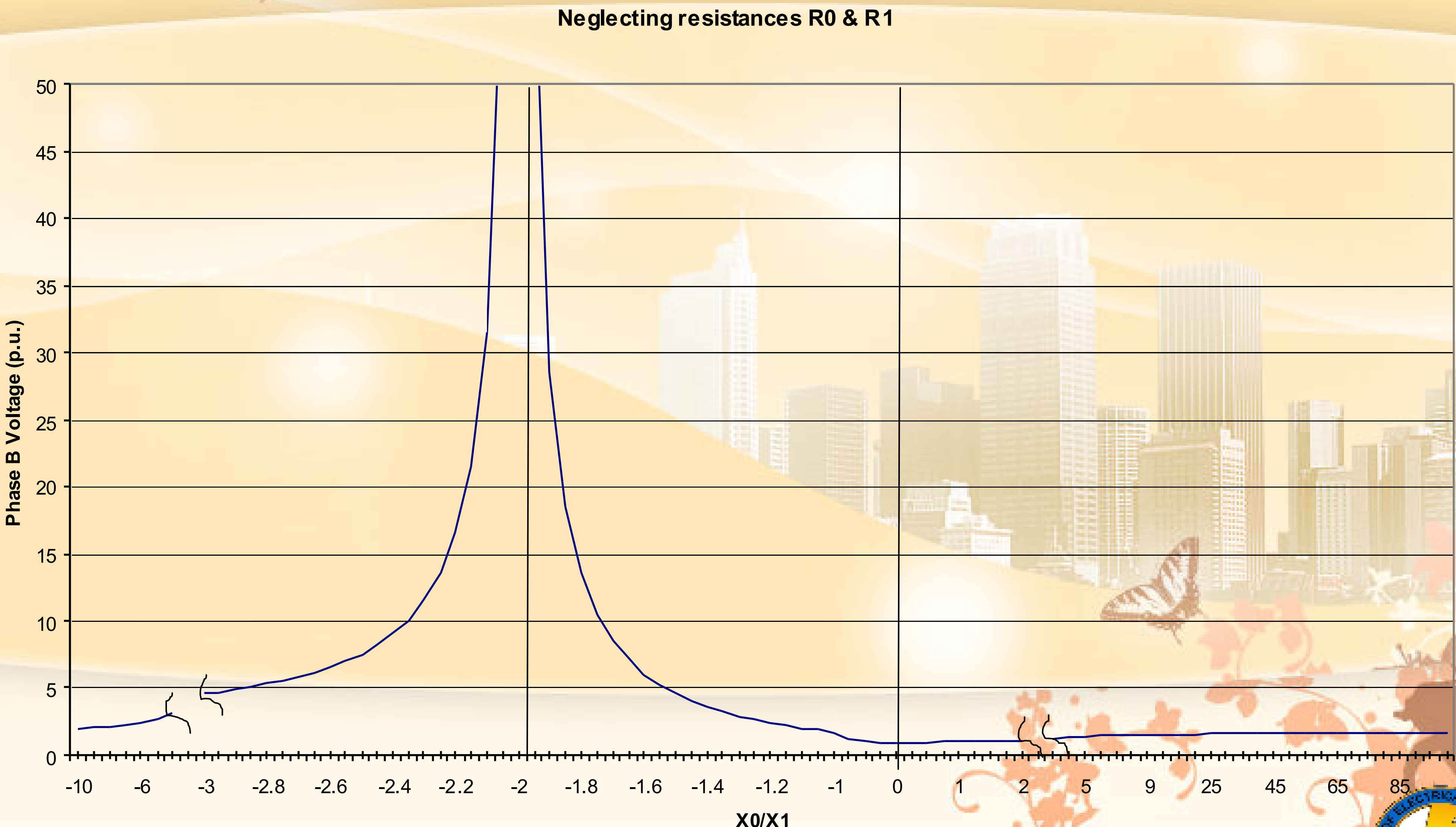
$$V_b = E_{a1}\left[a^2 - \left(\frac{Z_0 - Z_1}{2Z_1 + Z_0}\right)\right] = E_{a1}\left[a^2 - \left(\frac{Z_1}{Z_1}\right)\left(\frac{\frac{Z_0}{Z_1} - 1}{2 + \frac{Z_0}{Z_1}}\right)\right]$$

$$V_b = E_{a1}\left[a^2 - \left(\frac{\frac{Z_0}{Z_1} - 1}{2 + \frac{Z_0}{Z_1}}\right)\right]$$

neglecting resistances, R_1 and R_0;

$$V_b = E_{a1}\left[a^2 - \left(\frac{\frac{X_0}{X_1} - 1}{2 + \frac{X_0}{X_1}}\right)\right]$$

Unfaulted Phase B Voltage During Single Line-to-Ground Faults



Fault MVA

$$MVA_F = I_F \times MVA_{base}$$

where, for $E_{a1} = 1.0$ p.u.;

for three – phase fault in p.u. :

$$I_{F(3\phi)} = \frac{1}{Z_1}$$

for singe line - to - ground fault in p.u. :

$$I_{F(SLG)} = \frac{3}{Z_0 + 2Z_1}$$

Fault MVA

$$Three-phase\ \ fault\ \ MVA:$$

$$MVA_{F(3\phi)} = I_{F(3\phi)}(p.u.) \times MVA_{base}$$

$$Z_1 = \frac{1}{I_{F(3\phi)}}\ \ p.u.$$

$$Single\ \ line-to-ground\ \ fault\ \ MVA:$$

$$MVA_{F(SLG)} = I_{F(SLG)}(p.u.) \times MVA_{base}$$

$$2Z_1 + Z_0 = \frac{3}{I_{F(SLG)}}\ \ p.u.$$

$$Z_0 = \frac{3}{I_{F(SLG)}} - 2Z_1$$

Assumptions Made to Simplify Fault Calculations

1. Pre-fault load currents are neglected.
2. Pre-fault voltages are assumed equal to 1.0 per unit.
3. Resistances are neglected (only for 115kV & up).
4. Mutual impedances, when not appreciable are neglected.
5. Off-nominal transformer taps are equal to 1.0 per unit.
6. Positive- and negative-sequence impedances are equal.

Outline of Procedures for Short Circuit Calculations

1 Setup the network impedances expressed in per unit on a common MVA base in the form of a single-line diagram
2 Determine the single equivalent (Thevenin's) impedance of each sequence network.
3 Determine the distribution factor giving the current in the individual branches for unit total sequence current.

Outline of Procedures for Short Circuit Calculations

4 Interconnect the three sequence networks for the type of fault under considerations and calculate the sequence currents at the fault point.

5 Determine the sequence current distribution by the application of the distribution factors to the sequence currents at the fault point

6 Synthesize the phase currents from the sequence currents.

Outline of Procedures for Short Circuit Calculations

7 Determine the sequence voltages throughout the networks from the sequence current distribution and branch impedances

8 Synthesize the phase voltages from the sequence voltage components

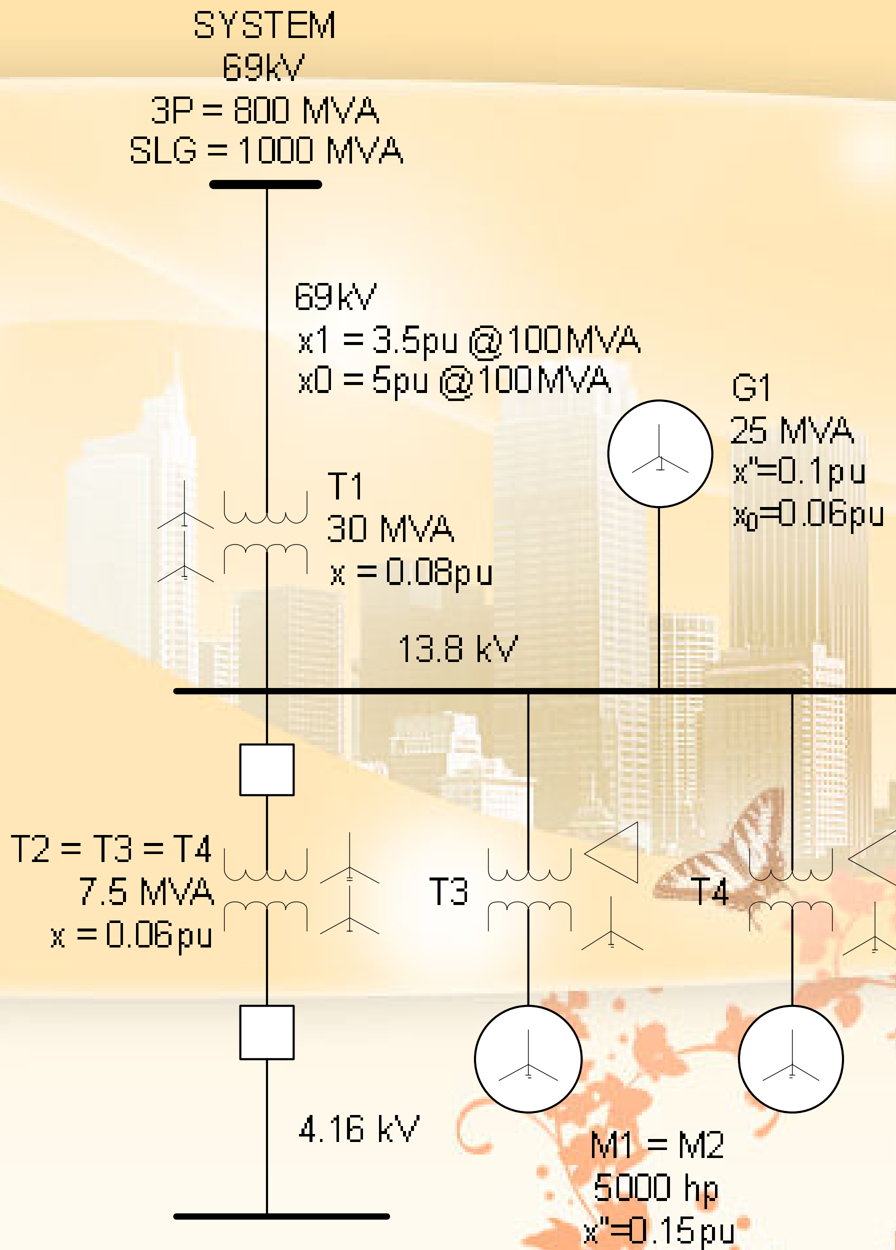
9 Convert the pre unit currents and voltages to actual physical units

CIRCUIT BREAKING SIZING (Asymmetrical Rating Factors)

- Momentary Rating
  - Multiplying Factor = 1.6
- Interrupting Rating
  - Multiplying Factor
    - 8 cycles = 1.0
    - 5 cycles = 1.1
    - 3 cycles = 1.2
    - 1 ½ cycles = 1.5

EXAMPLE PROBLEM

In the power system shown, determine the momentary and interrupting ratings for primary and secondary circuit breakers of transformer T2.



Solution:

Equivalent 69kV system@100MVA:

$$I_{F(3\phi)} = \left[\frac{800}{100}\right] = 8\,pu$$

$$I_{F(SLG)} = \left[\frac{1000}{100}\right] = 10\,pu$$

$$x_1 = \frac{1}{8} = 0.125\,pu$$

$$x_0 = \frac{3}{10} - 2 \times 0.125 = 0.05\,pu$$

T1:

$$x = 0.08\left[\frac{100}{30}\right] = 0.2667\,pu$$

G1:

$$x'' = 0.10\left[\frac{100}{25}\right] = 0.40\,pu$$

$$x_0 = 0.06\left[\frac{100}{25}\right] = 0.24\,pu$$

T2, T3, T3:

$$x = 0.06\left[\frac{100}{75}\right] = 0.80\,pu$$

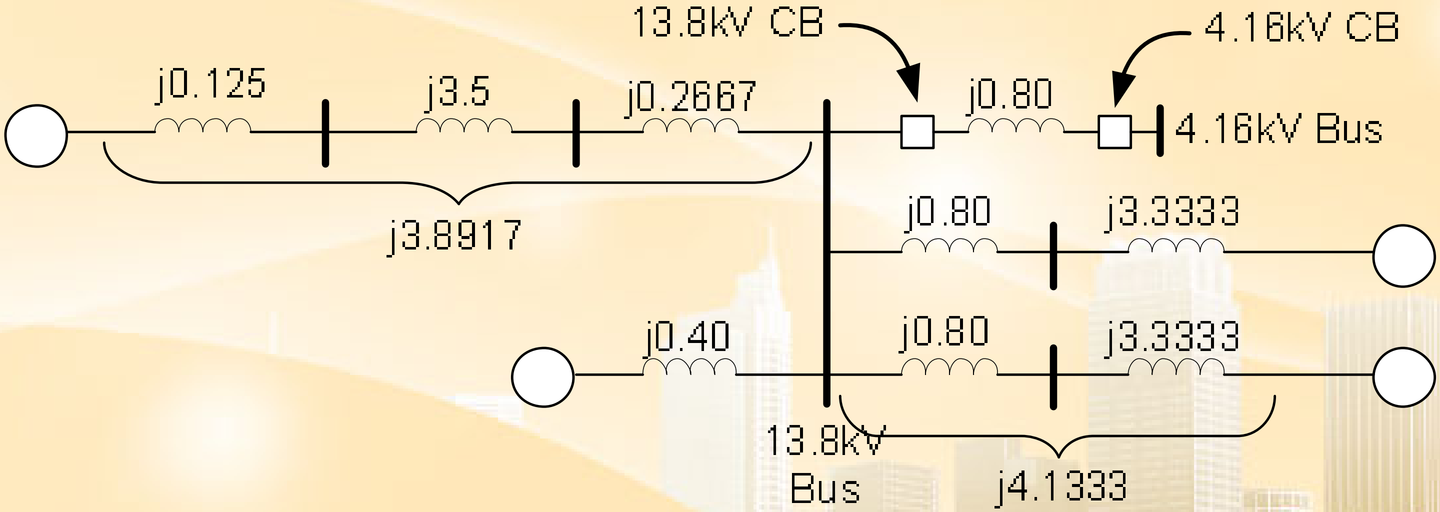
M1, M2: $kVA_B = 0.9 \times 5000 = 4500$

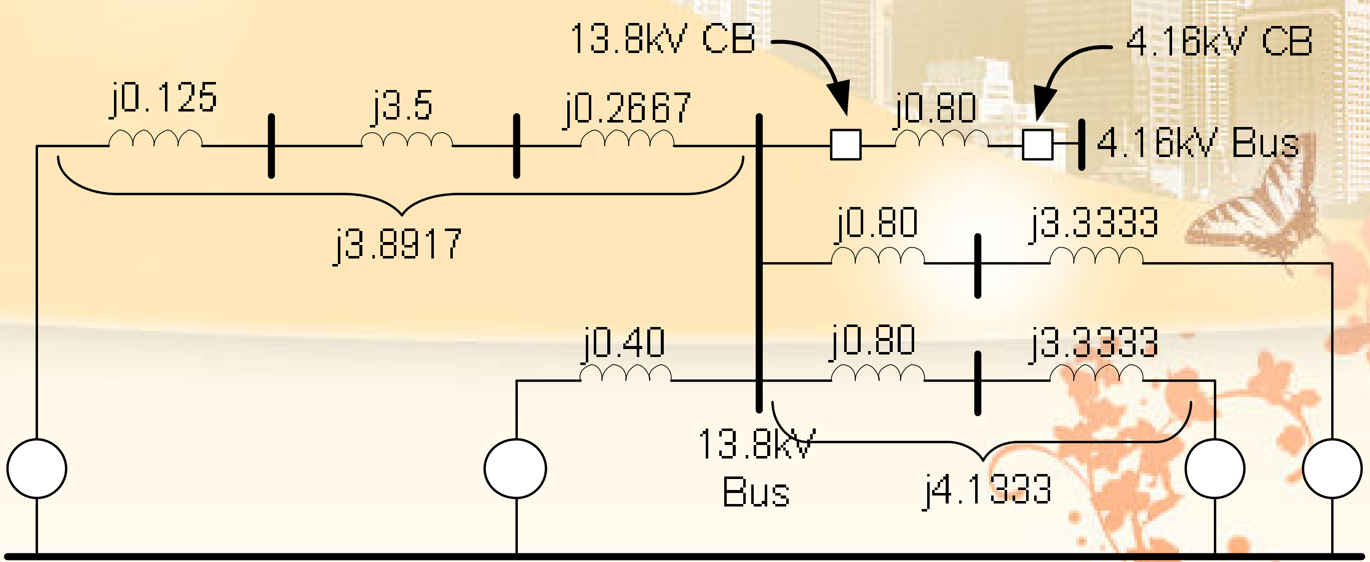
or $MVA_B = 4.5$

$$x = 0.15\left[\frac{100}{4.5}\right] = 3.3333\,pu$$

Solution:

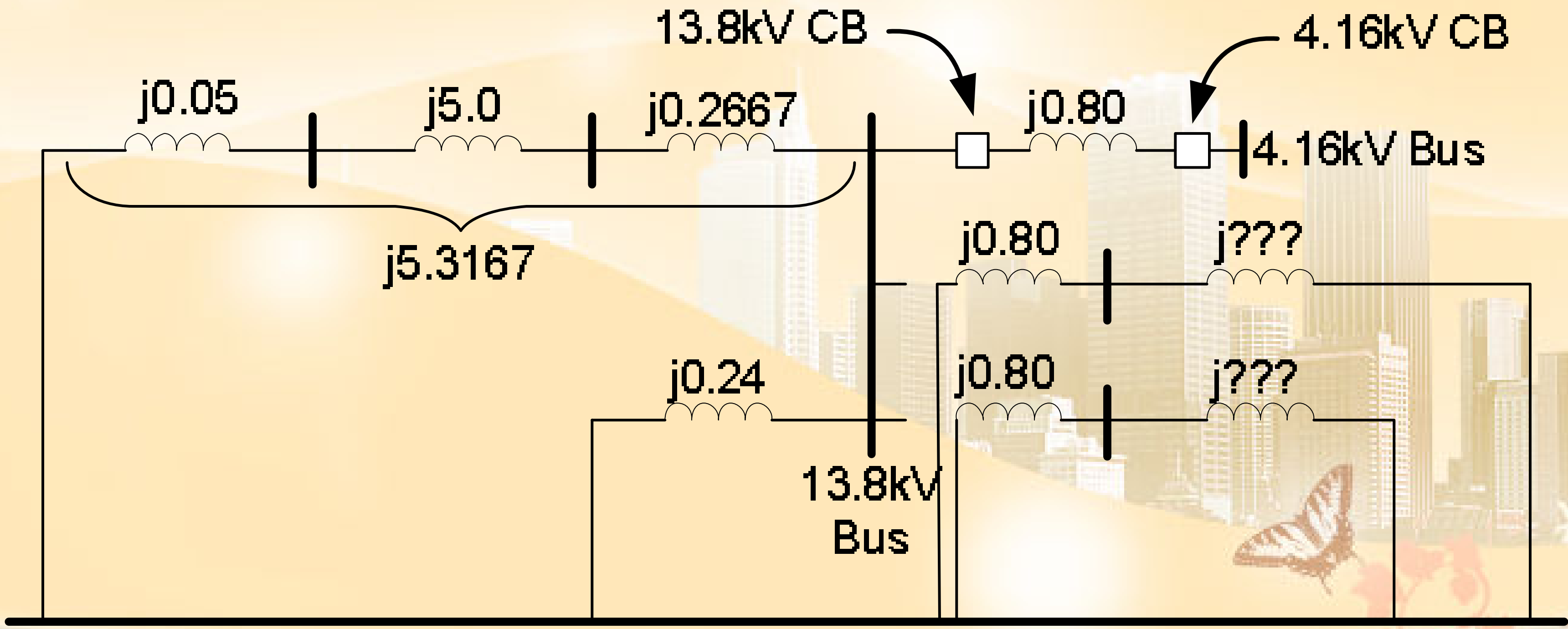
Positive sequence network:



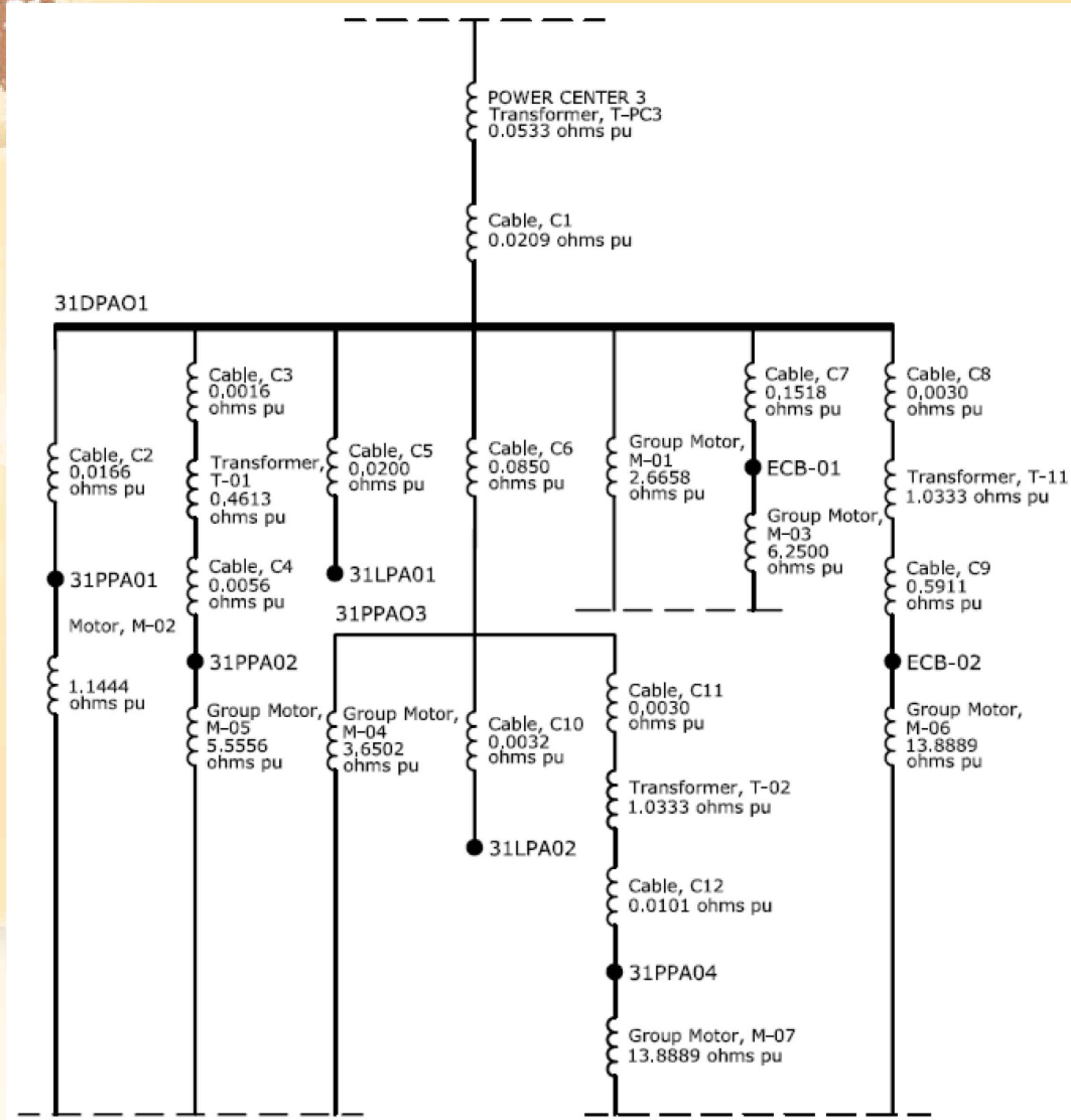


Solution:

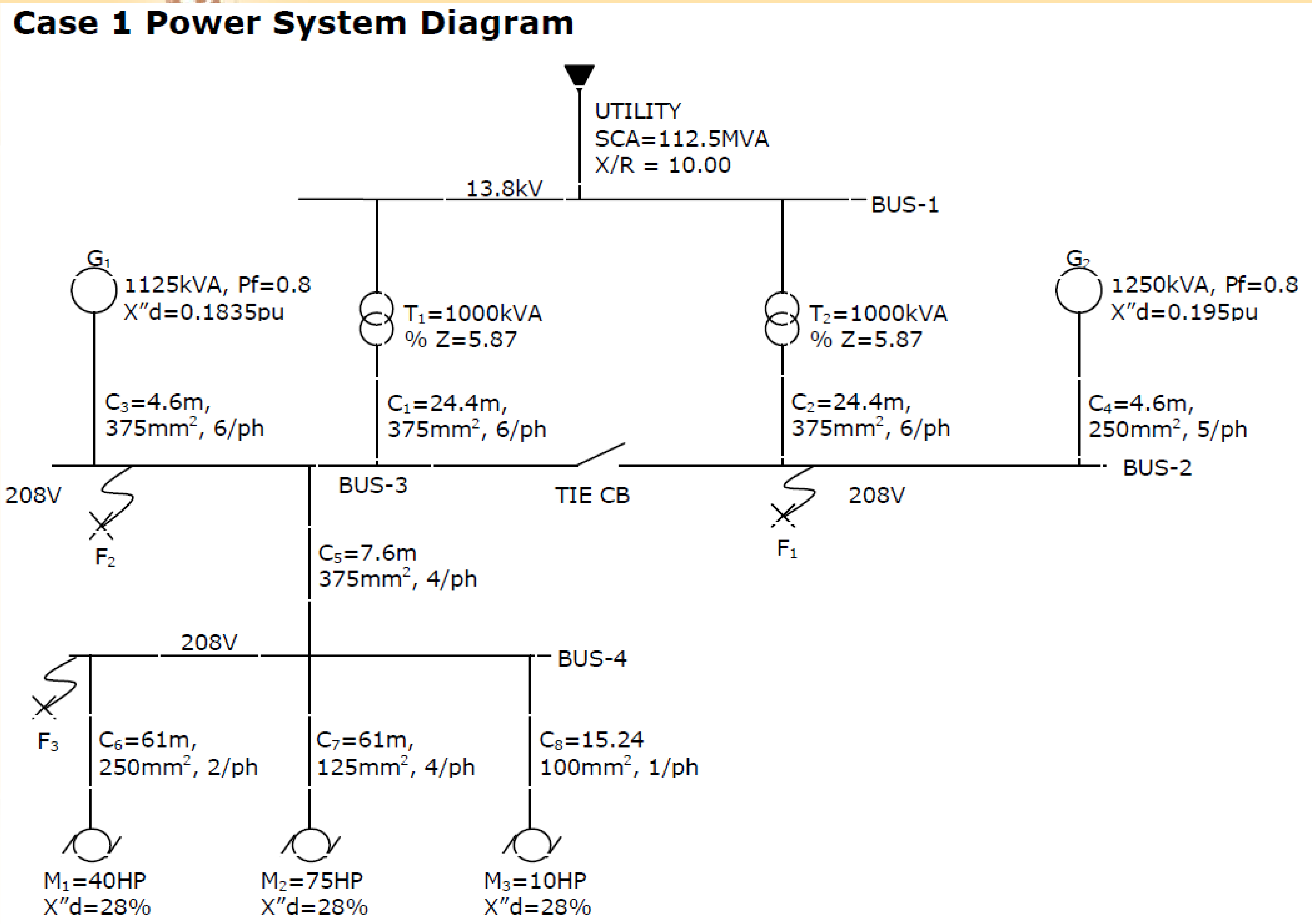
Zero sequence network:



Improper Sequence Network Models

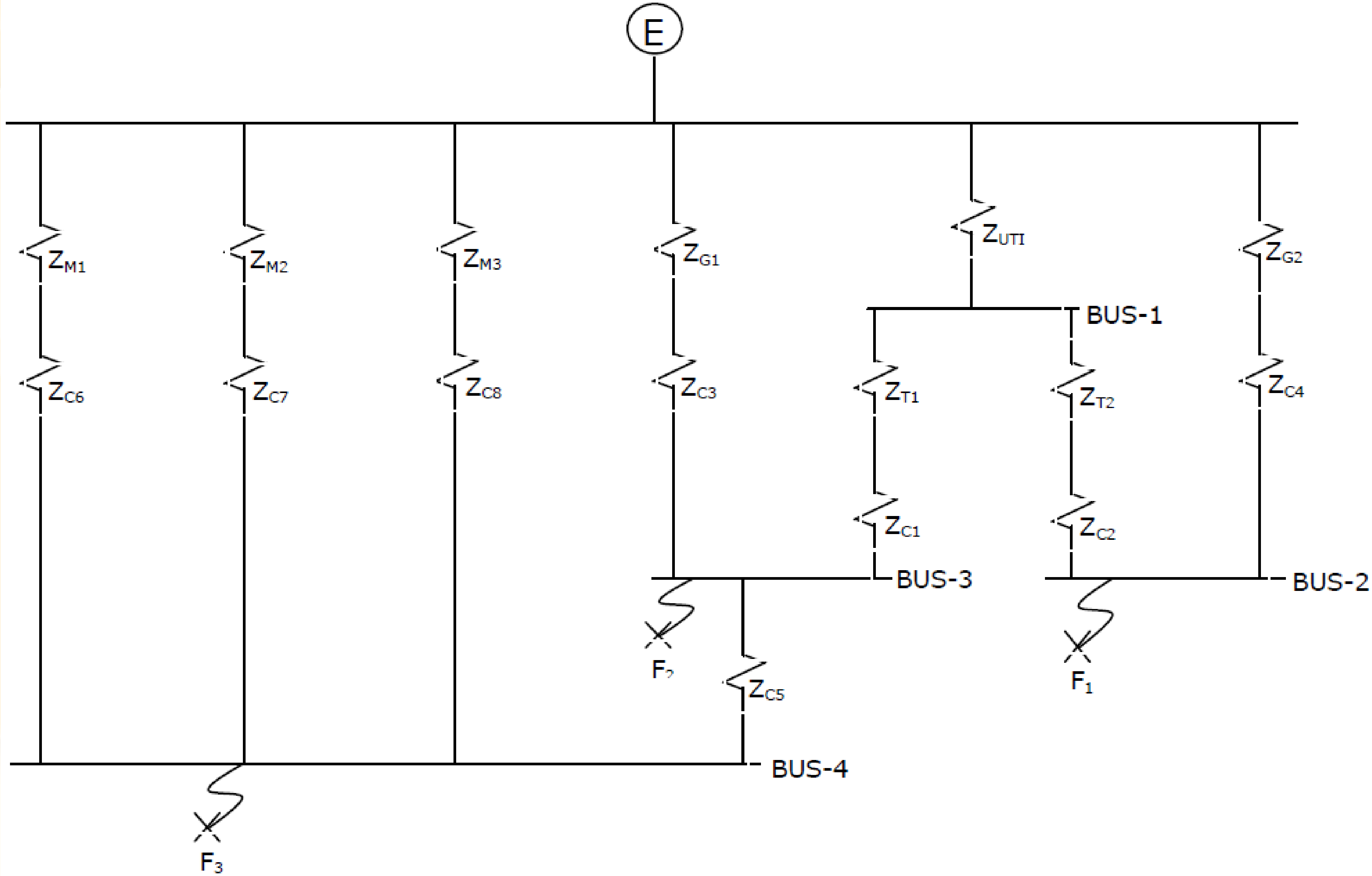


Improper Sequence Network Models

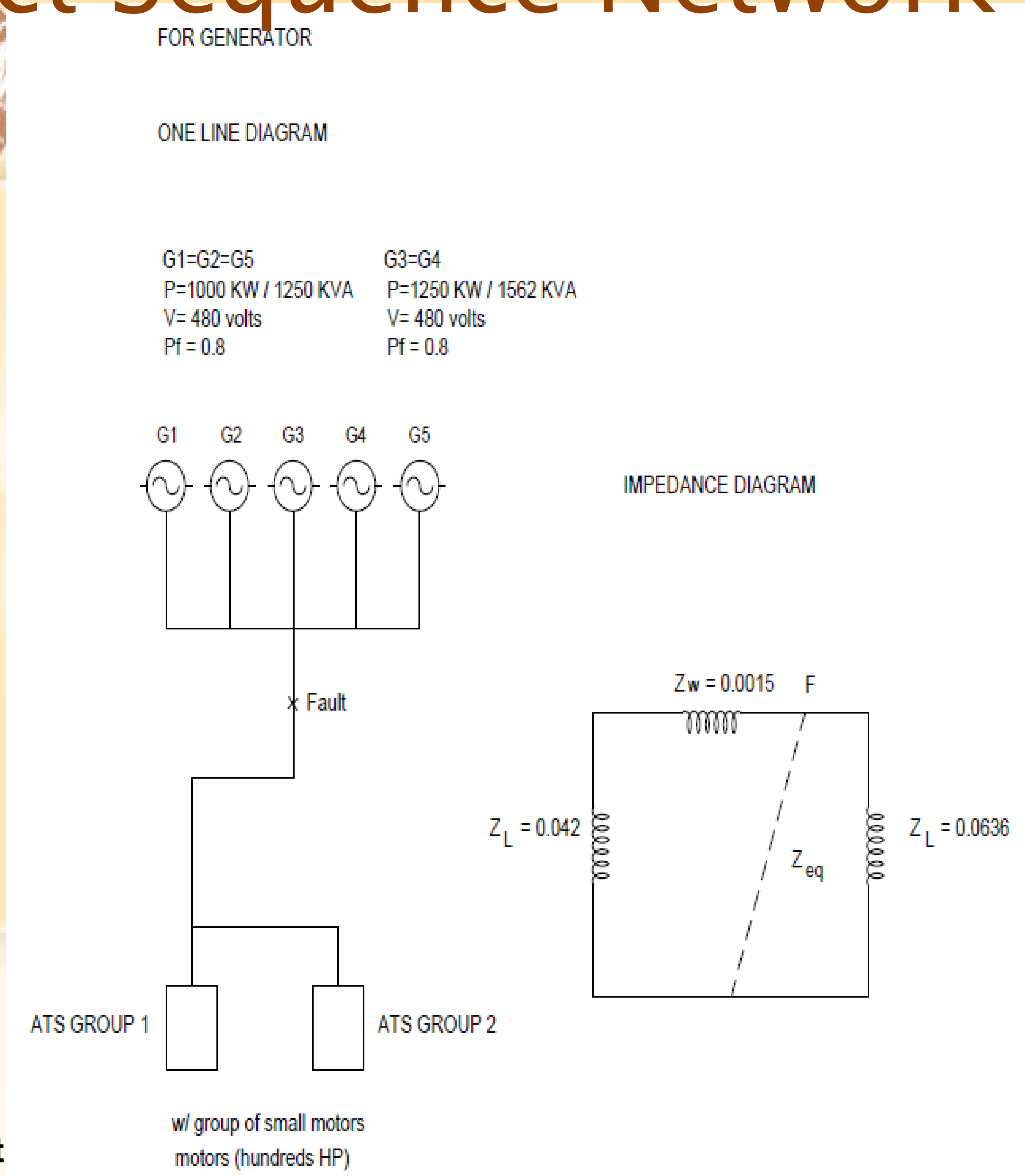


Improper Sequence Network Models

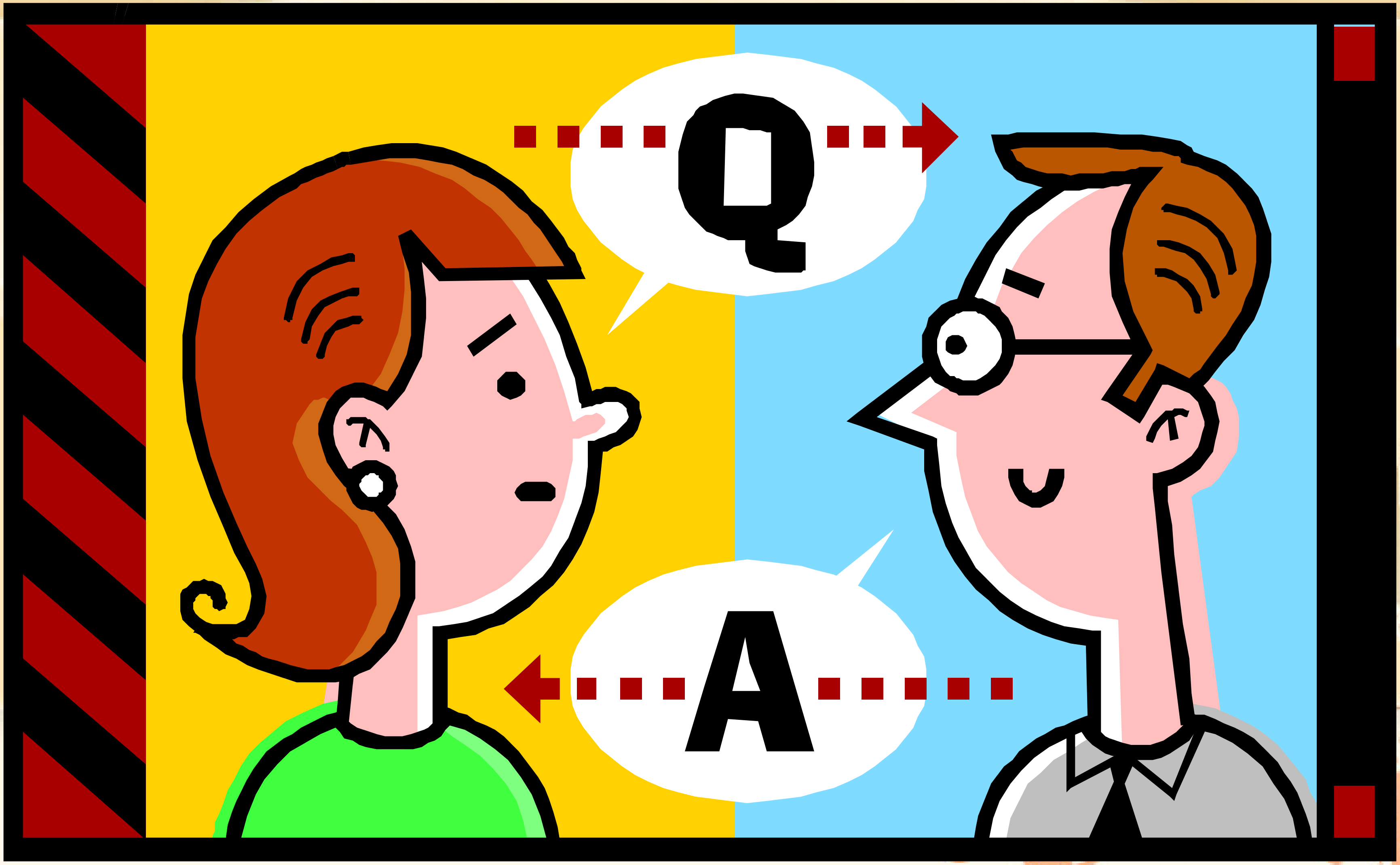
Case 1 – Impedance Diagram



Incorrect Sequence Network Models



QUESTIONS?



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